FAULT DETECTION OF INDUCTION MOTORS DUE TO THE EFFECTS OF MAGNETIC SATURATION

Abstract: In this paper, the impact damage to the motors on the course of zero-sequence voltage and its spectrum is presented. Detection signals results from detail analysis of the formula describing this voltage component is induced in the stator windings due to core magnetic saturation and the discrete displacement of windings. Its course is affected by operating both the stator and the rotor. Other fault detection methods, by analysing the spectrum of stator currents, are known and widely applied. The presented method is under investigation, but may be a complement to the other because of easy measurement of the zero voltage for star connected motors. The results of the method are presented by measurements and explained using mathematical modelling for the slip-ring induction motor.

Keywords: fault detection, induction motors, magnetic saturation

1. Introduction
The non-invasive methods of detection electrical and mechanical damages of induction motors are based mainly on the stator current spectrum analysis. The signals indicating different damages were separated from the current spectrum during a long process of investigation by many researchers. Here can be mentioned the methods developed from mathematical modeling [4, 5, 8] and the methods based on laboratory tests of many induction machines [6, 9] and the physical interpretation of obtained results. All the researches created a data-base that can be used for training the artificial intelligence detection systems [7].

The main subject of this paper is to add another detection signal that carries the information indicating the damages. This is the zero sequence voltage that is excited in the phase windings as a result of magnetic saturation [9, 10]. The waveform of this voltage incorporates the state of the air-gap magnetic field. It depends on the stator and rotor cooperation and also on the damages of them [9]. For studies the slip-ring induction motor was taken and the influence of the rotor circuit asymmetry was analyzed. During laboratory tests some accompanying phenomena appeared that had to be explained.

The scheme of the laboratory stand is shown in Fig. 1. The zero sequence voltage results from the measured neutral voltage \( u_n \) between the neutral point of the supplying transformer or the pseudo neutral point and the star point connection of the stator windings.

![Fig. 1. Connection of the slip-ring induction motor for tests of rotor asymmetry](image)

2. Mathematical background for induction motor diagnostics
2.1 Main assumptions for the mathematical model
- Magnetic core is saturated only due to the first non-zero symmetrical components of stator and rotor currents.
- The effect of saturation is simulated as enlarging of the air-gap length in the place \( \alpha_M \) where the total magneto-motive force (MMF) of the machine has the maximum with respect to the stator phase \( U \) assumed as the first and is represented by the magnetising current \( i_M \). This varying air-gap is expressed by per unit permeance function approximated with harmonic series of the form utilising Euler’s identity:
\[
N_{ou}^\mu (x, \alpha_M) = \frac{1}{2} \sum_{\mu} \Lambda_\mu (i_M) e^{j\mu p(x-a_u)};
\]

\[
\mu = 0, \pm 2, \pm 6, \pm 10, \pm 14, \ldots
\]

The reference permeance \( \Lambda_o = \frac{4\mu_0 \mu r_c}{\pi p^2 s} \), where

\[
\mu_o = 4\pi \cdot 10^{-7} \text{ H/m},
\]

\( r_c \) - inside stator radius, \( l_c \) - equivalent length of the machine core, \( s \) - air-gap equivalent length incorporating the Carter’s factor, \( p \) - pole pair number of the machine, \( x \) - angle along the air-gap of the machine cross section with respect to phase \( U \).

- The effect of slotting is modelled as a signal modulating the previously defined permeance function.
- The winding inductances are calculated using formulae for the machine with a non-uniform air-gap [1]. The machine model is formulated as a connection of resistances and inductances.
- The machine voltage equations are transformed from the natural system to symmetrical components. This transformation splits the harmonic components of inductances assigning them to voltage equations accompanying the respective symmetrical components.

The resultant permeance takes the general form:

\[
N_{ou}^\mu (x, \alpha, \varphi, i_M) = \frac{1}{2} \sum_{\mu} \sum_{\mu m} \Lambda_{\mu m, n} (i_M) e^{-j\mu a} e^{jmx} e^{j\varphi}
\]

\[
\alpha = p a_M
\]

Sets of harmonic orders for the permeance function:

\[
H_\mu = \{ \mu = 2i \; ; \; i = 0, \pm 1, \pm 3, \pm 5, \ldots \}
\]

\[
H_m = \{ m = kZ_s + lZ_r + \mu p \; ; \; k = 0, \pm 1, \pm 2, \ldots $$ \text{and} \; l = 0, \pm 1, \pm 2, \ldots \}
\]

\[
H_\varphi = \{ n = -lZ_r \; ; \; l = 0, \pm 1, \pm 2, \ldots \}
\]

Without slotting is \( m = \mu p \) and \( n = 0 \). Mutual inductance of two windings \( a \) and \( b \) positioned with respect to the stator at \( x_a \) and \( x_b \) respectively, is given by the function:

\[
M_{x/r/sr}^{ab} = \frac{1}{2} \sum_{\mu \in H_\mu} \sum_{m \in H_m} \sum_{n \in H_\varphi} M_{x/r/sr}^{\mu m, n} e^{-j\mu a} e^{-jnx_a} e^{jnymx} e^{j\varphi}
\]

where

\[
\forall \nu \in H_\nu \land \forall m \in H_m \Rightarrow \rho = -(\nu + m) \in H_\rho
\]

\[
M_{x/r/sr}^{x/r/s} = \frac{1}{2} \sum_{\mu \in H_\mu} \sum_{m \in H_m} \sum_{n \in H_\varphi} M_{x/r/s}^{\mu m, n} e^{-j\mu a} e^{-jnx_a} e^{jnymx} e^{j\varphi}
\]

Thus, the mutual inductance exists when the following condition is satisfied for the assumed harmonic orders \( \nu \in H_\nu, \rho \in H_\rho, m \in H_m \):

\[
\nu + \rho + m = 0
\]

This assures the harmonic balance inside this mathematical model.

For symmetrically designed windings the MMF harmonics have the orders belonging to the sets:

\[
H_\nu = \{ (2i - 1)p ; i = 0, \pm 1, \pm 2, \ldots \}
\]

for winding \( a \),

\[
H_\rho = \{ (2i - 1)p ; i = 0, \pm 1, \pm 2, \ldots \}
\]

for winding \( b \).

The phase winding positions for the 3-phase slip-ring induction motor assume values:

- for the stator

\[
x_a = (a - 1)\beta_s + x_b = (b - 1)\beta_s ;
\]

\[
a = 1,2,3; b = 1,2,3; \beta_s = \frac{2\pi}{3p}
\]

- for the rotor

\[
x_a = (a - 1)\beta_r + \varphi + x_b = (b - 1)\beta_r + \varphi ;
\]

\[
a = 1,2,3; b = 1,2,3; \beta_r = \frac{2\pi}{3p}
\]

\( \varphi \) - rotor rotation angle.

### 2.2 Voltage equations of the motor

The voltage equations of the motor are described as:

\[
\begin{bmatrix}
\begin{bmatrix}
  u_1^s \\
  u_1^r
\end{bmatrix}
\end{bmatrix}
= \begin{bmatrix}
  R_1^s & jL_1^s \\
  R_1^r & jL_1^r
\end{bmatrix}
\begin{bmatrix}
  i_1^s \\
  i_1^r
\end{bmatrix}
+ \frac{d}{dt}
\begin{bmatrix}
  M_1^s \begin{bmatrix}
  M_1^s & M_1^r \\
  M_1^r & M_1^s
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
  i_1^s \\
  i_1^r
\end{bmatrix}
\]

Vectors of voltages and currents contain phase voltages \( u_U, u_V, u_W \) and phase currents \( i_U, i_V, i_W \) for the stator and phase voltages \( u_K, u_L, u_M \) and phase currents \( i_K, i_L, i_M \) for the rotor. The matrices of resistances and leakage inductances are diagonal with different elements for each phase winding:
After transformation to symmetrical components the structure of equations remains the same as (12) with the superscript II indicating the vectors and matrices that now are described with complex numbers:

\[
u^I = [u_x^I, u_y^I, u_z^I]^T; \quad i^I = [i_x^I, i_y^I, i_z^I]^T
\]

The matrices of resistances \( R_{\alpha I}, R_{\beta I} \) and leakage inductances \( L_{\alpha I}, L_{\beta I} \) have the same general structure:

\[
Z^{II}_{\alpha} = \begin{bmatrix}
Z_0 & Z^* & Z \\
Z & Z_0 & Z^* \\
Z^* & Z & Z_0
\end{bmatrix};
\]

\[
Z_0 = Z_{x1} + Z_{x2} + Z_{x3}, \quad Z = Z_{x1} + aZ_{x2} + a^2Z_{x3}, \quad Z = R \text{ or } L; \quad x = s \text{ or } r \text{ or } sl \text{ or } rl
\]

The matrices of self and mutual inductances have the following structures:

\[
M^{II}_{\alpha} = \frac{1}{2} \sum_{\mu \in H_{\alpha}} \sum_{v \in H_{\alpha}} \sum_{m \in H_{\alpha}} \sum_{n \in H_{\alpha}} M_{\alpha m}^{\alpha} e^{-j\alpha u e^{m+n} Q_s},
\]

\[
M^{II}_{\beta} = \frac{1}{2} \sum_{\mu \in H_{\beta}} \sum_{v \in H_{\beta}} \sum_{m \in H_{\beta}} \sum_{n \in H_{\beta}} M_{\beta m}^{\beta} e^{-j\beta v e^{m+n} Q_r},
\]

\[
M^{II}_{sr} = \frac{1}{2} \sum_{\mu \in H_{sr}} \sum_{v \in H_{sr}} \sum_{m \in H_{sr}} \sum_{n \in H_{sr}} M_{sr m}^{sr} e^{-j\mu v e^{m+n} Q_{sr}}.
\]

Matrices \( Q_s, Q_r, Q_{sr} \) of dimension 3×3 have elements \( Q_{s/r/sr} \) equal to 1 or 0. The element \( Q_{s/r/sr} = 1 \) if for given harmonic orders \( \nu \in H_{\nu}, \rho \in H_{\rho} \), associated with the row, and \( \rho \in H_{\rho} \), associated with the column, the condition for harmonic order \( m = -\nu - \rho \in H_{m} \) is satisfied. This harmonic condition is indicated as a subscript of 1 in the matrix structure. Thus, the general form is

\[
Q_{\nu \rho \sigma} = \begin{cases}
1 & \text{if } \nu = -m, -m+1, \ldots, -1, 0, 1, \ldots \text{ and } \rho = -\nu-2, -\nu-1, 0, 1, \ldots \\
0 & \text{otherwise}
\end{cases}
\]

(13)

In the above:

\[
u^{(2)} = u_s^{(2)}, u_r^{(2)} = u_s^{(2)*}, l_s^{(2)} = l_s^{(2)*}, l_r^{(2)} = l_r^{(2)*}
\]

The real and imaginary parts are denoted \( \alpha \) and \( \beta \) respectively (stationary reference frame \( \alpha, \beta \) of symmetrical components, that are the space vectors), \( x \) - complex number conjugated of \( x \), superscript \( T \) means matrix transposition.

Both the equation sets for the stator and the rotor can be transformed into one reference frame \( x-y \) rotating at the angular speed \( \omega_x = \frac{d\alpha}{dt} \) with respect to the axis \( \alpha \) of the stationary reference frame \( \alpha, \beta \). The transformation matrix \( T_{xy} = \text{diag}[1, e^{j\alpha}, e^{j2\alpha}] \) does not change the zero component of stator and rotor voltages and currents. Denoting the real and imaginary parts of transformed voltage and current vectors with \( x \) and \( y \) respectively the following relationships are valid

\[
u_x = u_x + j u_y = u_x^{(1)} e^{-ja}
\]

\[
u_r = u_{rx} + j u_{ry} = u_{rx}^{(1)} e^{-ja}
\]

\[
i_x = i_x + j i_y = i_x^{(1)} e^{-ja}
\]

\[
i_r = i_{rx} + j i_{ry} = i_{rx}^{(1)} e^{-ja}
\]

\[
i_M = i_M + i_x = i_M^{(1)} e^{-ja}
\]

\[
i_y = -i_y
\]

(14)

Removing \( i_y \) from the set of differential equations the position angle \( \alpha \) becomes the state variable together with \( i_{xs}, i_{sy}, i_{rs} \) and the rotor speed \( \omega_x \).
From (13) it is clear that generally

\[ H_m = \{ m = 2pk; k = 0, \pm 1, \pm 2, \ldots \} \quad (15) \]

So, instead of matrix \( Q_{\text{m,sr}} \) the Table 1 of harmonic orders presenting the balance of \( \nu, \rho, m \) can be utilised. The threes of these numbers indicate the harmonic inductances \( M_{v,r}^{Jr/sr} \) prescribed to the given matrix cell.

Table 1. Balance of harmonics \( \nu, \rho, m \)

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( \rho )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pm 9p, \pm 3p, \ldots )</td>
<td>( \pm 7p, \pm 5p, \ldots )</td>
<td>( \pm 5p, \pm 3p, \ldots )</td>
</tr>
<tr>
<td>( m = 6pk )</td>
<td>( m = -2p(1+3k) )</td>
<td>( m = 2p(1+3k) )</td>
</tr>
<tr>
<td>( m = 2p(1+3k) )</td>
<td>( m = 6pk )</td>
<td>( m = -2p(1+3k) )</td>
</tr>
<tr>
<td>( m = -2p(1+3k) )</td>
<td>( m = 2p(1+3k) )</td>
<td>( m = 6pk )</td>
</tr>
</tbody>
</table>

2.3 Zero sequence voltage

The zero sequence voltage is described by the expression

\[ u_s^{(0)} = 2 \text{Re} \left\{ R_s^* L_s^t e^{j\alpha t} \right\} + \left[ 2 \text{Re} \left\{ L_s^t \frac{dI_s}{dt} e^{j\alpha t} \right\} + \frac{1}{2} \left\{ \sum_{\nu=H_s} \sum_{\rho=H_s} \sum_{m=H_s} j[n\omega - (\mu - 1)\omega] \left( \frac{M_{sr}^{\nu,\rho}}{c_{nu}} L_s^s e^{j(v+m)\varphi} \right) \right\} \right\} + \frac{1}{2} \left\{ \sum_{\nu=H_s} \sum_{\rho=H_s} \sum_{m=H_s} j[n\omega - (\mu - 1)\omega] \left( \frac{M_{sr}^{\nu,\rho}}{c_{nu}} L_s^s e^{j(v+m)\varphi} \right) \right\} + \frac{1}{2} \left\{ \sum_{\nu=H_s} \sum_{\rho=H_s} \sum_{m=H_s} (M_{sr}^{\nu,\rho} \frac{dI_s}{dt} + M_{sr}^{\nu,\rho} \frac{dI_s}{dt} e^{j(v+m)\varphi} \right\} e^{-j(\mu - 1)\omega t} e^{j\varphi t} \right\} \right\} \]  

\[ \omega_s = 2\pi \frac{f_s}{p} \quad \text{angular frequency of mono-harmonic supplying voltages.} \]

The waveform of zero sequence voltage \( u_s^{(0)} \) contains alternating components of frequencies specific for symmetry and indicating the asymmetry.

- Symmetry

There are two series

\[ \omega_s = -(\mu - 1)\omega_s + n\omega \]

\[ \omega_s = \omega_s + (v + m)p\omega \]

Incorporating \( \omega = \frac{2\pi}{p} (1 - s) \), \( \mu = -10, -6, -2, 0, 2, 6, 10, n = -IZ, v + m = (1 + 6k) p \) the following frequencies can be distinguished:

\[ f_s = f_s \left[ -(\mu - 1) - \frac{IZ}{p} (1 - s) \right] \]  

\[ f_{s2} = f_s \left[ -(\mu - 1) + \frac{IZ}{p} + (1 - s) (1 - s) \right] \]

\[ l = 0, \pm 1, \pm 2, \ldots; k = 0, \pm 1, \pm 2, \ldots \]

- Asymmetry

The relevant spectrum contains frequencies \( f_{s1} \) (18), \( f_{s2} \) (19) and the additional indicating the asymmetry:

\[ f_{s3} = f_{s1} - (n_1 + 1)sf_s \left\{ f_{s1} - 2sf_s \right\} \]

\[ f_{s4} = f_{s2} - (n_1 + 1)sf_s \left\{ f_{s2} - 2sf_s \right\} \]

\[ f_{s5} = f_s \]

3. Laboratory tests

The influence of rotor asymmetry on the zero sequence voltage was tested using the slip-ring induction motor SZUDe48a. The ratings are:

\[ P_N = 2.2 \text{ kW}, U_{e,n} = 380 \text{ V (Y)}, I_{e,n} = 7.6 \text{ A}, f_N = 50 \text{ Hz}, n_N = 690 \text{ rev/min}, \eta_N = 68 \%, U_{e,n} = 62 \text{ V (Y)}, I_{e,n} = 24.5 \text{ A}. \]
Stator parameters: \( m = 3 \) (number of phases), \( Z_s = 36, Z_r = 24, p = 4, q_s = Z_s/(2m_p) = 1.5 \).

The motor was supplied from the 3-phase mains of r.m.s. value 400 V. The supplying voltages were distorted with respect to pure sinusoid with higher harmonics of orders: 3 (6.85 V), 5 (3.84 V), 7 (1.01 V), 9 (0.892 V), 11 (1.413 V) 15 (1 V). The steady state waveforms are shown in Fig. 2.

The influence of rotor asymmetry was analyzed on the background of symmetrical operation. The asymmetry was performed as the break of the rotor phase \( M \). Results of measurements are shown in Figs. 3 and 4 for the motor loaded with approximately rated torque. Harmonic bars of \( u_n \) observed at symmetrical and asymmetrical operation can be predicted using formulae (19) and (20) and (21), (22).

As it is apparent the theoretically predicted harmonic components of \( u_n = u^{(0)}_n / \sqrt{3} \) specific for the symmetrical operation appear in the real machine. The harmonic bars for \( f_{s1} \) in Fig. 4a are caused by saturation. Additionally there is visible the bar for \( f_{s5} = 50 \) Hz indicating an asymmetry in stator. This can be caused by asymmetrical supply, internal asymmetry of the stator circuit or the anisotropy. For the studied motor the two last reasons are the most probable.
Rotor asymmetry is synonymously indicated by the bar $f_{s}$ in Fig. 4b. Additionally for currents $i_U$ and $i_K$ the characteristic bars have been shown for frequencies in square boxes. The distortion of supply voltages with higher harmonics does not influences the waveform $u_n$.

4. Conclusions
Magnetic saturation of induction motor generates new signals of the neutral voltage indicating asymmetry of the rotor circuit. This is profitable only for the star connected stator winding. For the delta connection the zero sequence stator current could be taken into account. Other damages like eccentrically rotating rotor can be detected using this method also. Distortions of supplying voltages influence $u_n$ in a very low degree. So, this method could be applied for fault detection of induction motors supplied with power electronics.

5. Bibliography


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