Abstract: The work presented in this paper relates to the Interior Permanent Magnet Synchronous Motor (IPMSM) optimization procedure programmed in Matlab and Maxwell environments. The stator of the machine is an mass-produced one with concentrated winding. In the first optimization stage geometry of IPMSM machine was considered, concerning average torque value maximization and cogging torque minimization with physical and technological constraints. By combining Matlab software and Maxwell application authors used genetic algorithm for Finite Element Model optimization. Moreover $L_d$ and $L_q$ inductances were estimated for evaluation of CPSR machine capabilities and selection for the best geometry among Pareto Front solutions.

1. Introduction

Thanks to the rapid advancement in the field of power electronics, digital signal processors and control algorithms PM excited and Switched Reluctance Motors are finding more and more applications and have replaced traction systems of most present hybrid electrical vehicles (HEVs) because they offer high performance over other DC and AC machines [1, 2, 3]. In particular, owing to development of rare-earth magnets with high energy product, it is possible to develop high power density machines with high overall efficiency. Furthermore, extended high speed capabilities, demanded in a highway cycle, are achieved thanks to proper rotor geometry design, and field weakening control strategies [3, 4, 5, 6]. Surface and radially-laminated Interior PM synchronous machines with conventional structure have limited or zero flux-weakening capability [5, 7]. Properly designed IPMSMs are capable of operating in CPSR (Constant Power Speed Region) – such machines perform also inverse saliency – that is $q$-axis inductance is larger than $d$-axis inductance. Consequently, it has an additive torque value, so called reluctance torque, that may be exploited to extend CPSR. Other positive feature of IPM rotor is that centrifugal forces cannot damage the magnet, thus whole construction is mechanically robust, especially for high speeds. To protect environment, there is a strong demand to develop highly efficient motors with high torque/mass ratio. Small cogging torque reduces noise and mechanical vibrations which cause increased reliability [8, 9, 10]. All these requirements can be fulfilled by appropriately designed PM motors [7, 11, 12]. Proposed geometry has segmented magnet poles oriented in the radial direction and iron bridges between magnets that provides additional flux canals to give the rotor inherent capability of flux weakening. Such structures are referred as Segmented IPM machines [5, 7]. Optimization procedure with genetic algorithm has been implemented in Maxwell and Matlab tools. The final geometry has been analyzed regarding inductances and air-gap magnetic flux density.
2. PMSM mathematical model
Torque developed in IPM motor can be divided into PM-caused component and reluctance torque as shown in Fig. 2. Mathematical model of IPM machines can be described as follows:

\[ T_m = \frac{3}{2} p_r \left[ \Psi_{pm} \cdot I_q + \left( L_d - L_q \right) \cdot I_d \cdot I_q \right] \]  

(1)

The first term in the equation (1) is the PM generated torque, and the second term is the reluctance torque which is proportional to the difference in stator inductances, \( L_d \) and \( L_q \). In the analyzed IPMSM, \( L_q \) is higher than \( L_d \) (due to the lower reluctance in \( q \)-axis direction), because the magnetic flux flowing along the \( d \)-axis has to cross through the magnet cavities in addition to the rotor air gap, while the magnetic flux of the \( q \)-axis crosses only the air gap. The \( d \)-axis is also basically magnetized with PM \([13, 14]\). Such difference increases torque and extends CPSR. Mathematical model for IPM machines is presented by following equation set:

\[
\begin{align*}
U_d &= R I_d + \frac{d\Psi}{dt} - p_r \Omega \Psi_q \\
U_q &= R I_q + \frac{d\Psi}{dt} + p_r \Omega \Psi_d \\
\Psi_d &= L_d I_d + \Psi_{PM} \\
\Psi_q &= L_q I_q
\end{align*}
\]

(2a) \( \quad \) (2b) \( \quad \) (2c) \( \quad \) (2d)

The control strategy applied for such motors meets several limitations:

\[
\begin{align*}
U_d^2 + U_q^2 &< U_N^2 \quad (3a) \\
I_d^2 + I_q^2 &< I_N^2 \quad (3b)
\end{align*}
\]

For the high speed regions flux from magnets gives high electro-motive force, which exceeds supply voltage. Using field weakening method, main flux is decreased by \( d \)-axis negative current (2c) and thus it is possible to stay in the voltage limit (3a). Optimum flux-weakening condition can be written as:

\[ I_{\text{opt}} = \frac{\Psi_{PM}}{L_d} \]

(4)

Such designs are called optimal field-weakening IPM motor designs and theoretically exhibit unlimited CPSR. Poles segmentation provides physical reduction of the air-gap magnet flux - \( \Psi_{PM} \) - during flux-weakening, thus very high ratio of \( L_q \) to \( L_d \) is not crucial to extend CPSR. Authors try to consider \( L_q/L_d \) ratio, and PM decrease in the high current regions.

3. Design problem
The case of study is represented by a 4-pole segmented IPMSM with fixed stator geometry and winding parameters. Rotor is equipped with NdFeB magnets \( (B_r = 1.23 \, \text{T}, \, 3x7x40 \, \text{mm}) \). Most important motor dimensions are presented in Table 1. Before the optimization process implementation, classical and segmented IPMSM geometries were considered. According to the literature \([4, 5]\) fractional magnets arrangement may provide very wide flux-weakening range with high overall performance parameters. In such structures \( d \)-axis current is still used for PM demagnetization but it is also used to alter PM flux-paths. The demagnetizing current causes some part of PM flux to be canalized into the rotor iron section between magnet poles. In such a way PM flux passing through the air-gap is efficiently reduced, while the PM-flux is mostly preserved.

Initial design has been pre-optimized with simple iterative process. Selected design variables have been chosen according to the geometrical constraints, and are presented in Table 2. Analyzed geometry design variables are also depicted in Fig. 1.

\[ \text{Fig. 1. Initial geometry cross-section} \]

Table 1. Machine parameters

<table>
<thead>
<tr>
<th>Rotor Outer Diameter [mm]</th>
<th>Air gap length [mm]</th>
<th>Stator Outer Diameter [mm]</th>
<th>Magnet Thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1</td>
<td>120</td>
<td>3.25</td>
</tr>
</tbody>
</table>
4. Optimization process

In the optimization procedure authors used heuristic technique based on struggle between individuals commonly known as Genetic Algorithm (GA). GA mimics natural selection process were the strongest individuals survive and non-important features fades away during struggle between genes – according to present demanding [12, 15]. Details of used genetic algorithm have been described in detail in [16].

4.1. First optimization stage

The problem presented in the paper is a Multiobjective Optimization Problem (MOP) subject to a set of constraints. For the proposed geometry the following objective functions should be minimized, and are defined as:

\[ f_1(x) = \max(T_{\text{avg}}(t)) \]  
\[ f_2(x) = -\text{mean}(T_{\text{em}}) \]

After solving the MOP a set of optimal non-dominated solutions were generated, creating Pareto front shown in Fig. 4. It is a set of models that acts as area for selection of best individual. The final selection should be made taking into account other features that the motor should have [1].

4.2. Second optimization stage

For obtained set of models \( L_d \) and \( L_q \) inductances values has been evaluated using method presented in [13]. It is based on Lagrange formalism, which, according to Sobczyk [17], may be applied to PM excited machines. A model with the highest ratio of \( L_q/L_d \) has been selected for the final analysis. Whole optimization algorithm has been presented in Fig. 2 and has been described in detail in [12, 15].

<table>
<thead>
<tr>
<th>( x_1 ) [mm]</th>
<th>( x_2 ) [mm]</th>
<th>( x_3 ) [mm]</th>
<th>( x_4 ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Barrier X position</td>
<td>Flux Barrier Y position</td>
<td>Flux Barrier Radius</td>
<td>Magnet Inset Radius</td>
</tr>
<tr>
<td>4-8</td>
<td>20-25</td>
<td>2-3</td>
<td>20-27</td>
</tr>
</tbody>
</table>

Table 2. Design variables constraints

5. Results

The direct problem has been solved using a 2D FE model of the motor; torque has been calculated with the virtual work principle. GA used 10 generations with 70 models in population. First optimization stage lasted for about 6 hours on typical performance PC (15, 8GB RAM, Windows 7). The mesh of the model consisted of a few thousand of elements.

Fig. 2. Optimization algorithm workflow

Fig. 3. Pareto front for optimal geometrie

In the second optimization stage all Pareto-Optimal geometries were compared considering \( L_q/L_d \) ratio. The final geometry is presented in Fig. 7 (\( L_q/L_d = 1.8 \)).
6. Summary

Maxwell and Matlab tools were combined for the rotor geometry optimization process of Segmented Interior Permanent Magnet Synchronous Motor. The connection between these two packages allowed flexible FE model geometry definition and analysis as well as effective results evaluation. The whole optimization process has improved average torque value significantly with the same cogging torque value prior to initial geometry. The simulation points out the benefits of the optimization using a genetic algorithm. Non ideal torque waveforms and higher harmonics (Fig. 6) in the air-gap magnetic flux density make sinusoidal phase currents improper for such IPM motors. Simple power supply system applied during simulations causes high torque ripples. It should be emphasized that Maxwell can be connected with Simplorer, therefore whole analysis of a drive system, may be properly evaluated [18] with cascade control structure commonly used in practical applications. During simulations there were encountered several problems. The first one was proper mesh settings in order to shorten whole optimization process, and assure correct FEM calculation results. Particularly, mesh should be more dense in the air gap region, and in the stator core could be thinner.
The second problem was to select proper control angle for each model to produce the highest torque from the available current. The control angle in each model was constant, thus torque evaluation exhibit minor error (current in each phase was sinusoidal with 1.6 A rms value). $L_d/L_q$ ratio for Pareto-optimal models varied slightly, but the ratio was not evaluated for other analyzed models. Accurate procedure needs proper power angle evaluation for each model during GA optimization which strongly extends calculation time with proposed inductance estimation procedure. Future work will be focused on $L_d$ and $L_q$ saturated parameters calculation, because nonlinear IPM motor characteristics may affect CPSR performance.

7. Literature

[18]. Palka R., Paplicki P., Piotuch R., Wardach M.: Simulation results of a permanent magnet synchronous motor with interior rotor which is controlled by the hysteresis current controller, Electrical Review, no. 02b/2013, pp. 147-149, (2013)
**Authors**
M.A. Rafal Piotuch, rafal.piotuch@zut.edu.pl
Prof. Ryszard Palka, rpalka@zut.edu.pl
West Pomeranian University of Technology
Szczecin, Department of Power Systems and Electrical Drives,
ul. Sikorskiego 37, 70-313 Szczecin,
tel.: +48 91 449 48 73

**Acknowledgment**
This work has been created with the support of the Ministry of Education and Science, Poland, under grant N N510 508040.