DISCREPANCIES OF THE EXPANDED PARK EQUATIONS AND PARAMETER IDENTIFICATION WITH EVOLUTION STRATEGY

Abstract: The Park equations are a well known analytic model for describing synchronous machines. Over the years they were consistently extended, like the additional Canay leakage inductance in the rotor circuit. Although, there are newer and more accurate analytic models, the Park equations are still applied by all manufacturers and grid operators. The expanded Park theory [1] derives these equations from the phase-domain model. In this paper at hand, it is shown that this derivation is based on false assumptions for calculating the inductances. Therefore the transformation of the phase-domain parameter does not lead to a decoupled system as it should in the expanded Park theory. It is shown that the new system is more complex and the parameter of this system could not be derived from the phase-domain model. Two different synchronous machines, one with a salient pole rotor and one with a round rotor, are modeled by a finite-element-method (FEM) software and all stator self and mutual inductances are calculated. The model of the round rotor machine is verified by measurements. Due to common application of the Park equations a new method to determine the parameter is also presented. The results of a three-phase sudden short circuit are used in combination with the evolutionary algorithm to derive the Park parameters. All optimized parameters are totally detached from the physics. The found parameters are verified by measurements of other transients, like the two-phase short circuit.

Keywords: electrical machines, Park, parameter identification, evolution strategy

1. Introduction
Due to the growth of renewable energy and as a consequence thereof fluctuating feed of electricity of wind or photovoltaics power station more and more transients occur in the grid. Hence, it is absolutely necessary to calculate transients and steady-state of synchronous machines correctly. Park’s equations [2] are commonly used as the analytical model for calculations of the synchronous machines and therefore for turbo generators. The difficult point is to determine those parameters. In the expanded Park theory, e.g. in [1] and [3], phase-domain parameters are transformed to the rotor fixed coordinate system of Park. In this paper at hand it is shown that the expanded Park theory depends on incorrect assumptions when calculating the required Park inductances from the phase-domain inductances. This leads to wrong parameter sets and inaccurate simulation results. Especially deviation of the currents are critical, as the electromagnetic forces of the machine are depended by the power of two on the stator currents.
A good option to determine parameters is the identification based on measurements. It is shown that the use of measurements combined with the evolutionary algorithm leads to good results.

2. Expanded Park theory of synchronous machines
The Park equations are the common analytical model for calculating synchronous machines. It is based on the transformation of the time- and rotor angle depended parameters and values to a rotor fixed coordinate system. The main benefit is that the inductances have constant values and that the mathematical system is decoupled.
The derivation of the expanded Park theory starts with the Euler-Lagrange equation:
\[
\frac{\partial H}{\partial q_k} - \frac{d}{dt} \frac{\partial H}{\partial \dot{q}_k} = 0
\]
where \(H\) is the Lagrangian and \(q_k\) the \(k\)-th generalized coordinate. The Lagrangian \(H\) of the considered system consists of the kinetic energy \(T\), the potential energy \(V\), the loss energy \(W_l\) and the input energy \(W_i\):
\[
H = T - V + W_l - W_i
\]
The generalized coordinates of a synchronous machine are the mechanical rotor angle \(\phi\) and the electric charges \(Q\) of each winding. Usually there are 3 stator windings, one field
winding and 2 damper windings, hence the number of the generalized coordinates is 7.

The needed energies of the machine can be found, for instance, in [1] and [4].

The result of the Euler-Lagrange equation (1) are six voltage equations and one mechanical equation:

\[
\begin{bmatrix} V_s \\ V_R \end{bmatrix} = \begin{bmatrix} [R_s] & [0] \\
[0] & [R_R] \end{bmatrix} \begin{bmatrix} I_s \\ I_R \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} [M_{SS}] & [M_{SR}] \\
[M_{RS}] & [M_{RR}] \end{bmatrix} \begin{bmatrix} I_s \\ I_R \end{bmatrix}
\]

(3)

and

\[
\frac{d}{dt} J\psi + D_R \frac{d\phi}{dt} = \frac{1}{2} \sum_{k=1}^{6} \sum_{n=1}^{6} \frac{\partial n_{kn}}{\partial \phi} i_k n + m_a
\]

(4)

The inductance matrices are full matrices where the elements of M_{SS}, M_{SR} and M_{RS} are dependent of the rotor angle \( \phi \).

\[
M_{SS} = \begin{bmatrix} L_{aa}(\phi) & L_{ab}(\phi) & L_{ac}(\phi) \\
L_{ba}(\phi) & L_{bb}(\phi) & L_{bc}(\phi) \\
L_{ca}(\phi) & L_{cb}(\phi) & L_{cc}(\phi) \end{bmatrix}
\]

(5)

The self inductances in eq. (5) are calculated in the expanded park theory by

\[
L_{aa}(\phi) = L_a + M_1 + M_2 \cdot \cos(2p\phi)
\]

\[
L_{ba}(\phi) = L_a + M_1 + M_2 \cdot \cos \left(2p \left(\phi - \frac{2\pi}{3}\right)\right)
\]

\[
L_{cb}(\phi) = L_a + M_1 + M_2 \cdot \cos \left(2p \left(\phi - \frac{4\pi}{3}\right)\right)
\]

(6)

and the mutual inductances by the example of \( L_{ca}(\phi) \):

\[
L_{ca}(\phi) = -\frac{M_1}{2} + M_2 \cdot \cos \left(2p \left(\phi - \frac{2\pi}{3}\right)\right)
\]

(7)

Both inductances, self and mutual, consist of a steady and an alternating component. The alternating component of the self inductances is equal to the alternating component the mutual inductances. Also the steady component of the self inductances minus the leakage inductance \( L_a \) is minus twice the steady component of the mutual inductances.

To derive Park’s equations of the phase-domain model, the transformation matrix \([T]\) is implemented [4]:

\[
[T] = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\
\cos(\phi) & \cos(\phi - \frac{2\pi}{3}) & \cos(\phi - \frac{4\pi}{3}) \\
-\sin(\phi) & -\sin(\phi - \frac{2\pi}{3}) & -\sin(\phi - \frac{4\pi}{3}) \end{bmatrix}
\]

(8)

After transforming the voltage equations, Park’s equations of the direct and quadrature axis are derived:

\[
V_q = R_s l_q + \frac{d}{dt} \left[ l_d l_a + L_{md} (l'_q + l_{da}) \right] - \frac{d}{dt} \left[ L_q l_q + L_{mq} l'_{dq} \right]
\]

(9)

and

\[
V_q = R_s l_q + \frac{d}{dt} \left[ l_d l_a + L_{md} (l'_q + l_{da}) \right] + \phi \left[ l_d l_a + L_{md} (l'_q + l_{da}) \right]
\]

(10)

with the mutual inductance \( L_{md}, L_{mq} \) of the d- and q-axis. They can be calculated directly from the inductances of the phase-domain model:

\[
L_{md} = \frac{3}{2} (M_1 + M_2)
\]

(11)

\[
L_{mq} = \frac{3}{2} (M_1 - M_2)
\]

Park’s equations result in the circuits shown in Fig. 1 and 2.

The mechanical equation (4) becomes after transformation

\[
\frac{d}{dt} J\psi + D_R \frac{d\phi}{dt} = \frac{3}{2} (\Psi_q l_q - \Psi_d l_d) + m_a.
\]

(12)

3. Analysis of inductances \( L_{md} \) and \( L_{mq} \)

As it results from chapter 2 the mutual Park inductances \( L_{md} \) and \( L_{mq} \) are coupled to the stator self and mutual inductances of the phase-domain model. Hence, it is necessary to make a closer look of these inductances.

For that reason the inductances of two synchronous machines are investigated. Thereby the classical hypothesis like no saturation or no eddy currents are regarded. Later the influence of the saturation is under investigation. The first machine is a round rotor synchronous machine with a rated power of 50 kW (Fig. 3a). The second machine has a salient pole rotor (Fig. 3b). So the focus lies on both different types of synchronous machines to get a comprehensive result. Both machines are
modeled with the finite-element software Ansys Maxwell 2D. The FE-model of the round rotor machine is verified with measurements. To consider possible influences of the saturation, the machines are modeled as well as with linear and with non linear magnetic material.

(a) Round rotor  (b) Salient pole rotor

Fig. 3: 2D cross section of the synchronous machines

To determine the inductances the distribution of the permeability is stored, a current is injected in one winding and the magnetic flux is evaluated in each winding. For instance, the self inductance of stator winding a and the mutual inductance between winding a and c is determined by:

\[
L_{aa} = \frac{\psi_a}{I_a} \quad L_{ca} = \frac{\psi_c}{I_a}
\]

Therefore it is possible to calculate each inductance of the phase-domain model. Due to the influence of the stator and rotor slots the fundamental wave of each inductance is derived by a fast Fourier transform (FFT). This is admissible since the Park theory also regards just the fundamental frequency.

The stator self and mutual inductance of both machines are depicted in Figs. 4 and 5. Based on a symmetrical stator geometry it is sufficient to consider just one stator self inductance and one stator mutual inductance. The other inductances distinguish in another angle but neither in the amplitude nor in the steady component.

3.1 Round rotor synchronous machine

The self inductance of the stators \( L_{aa} \) results in

\[
L_{aa} = 1.824 \cdot 10^{-3} + 3.13 \cdot 10^{-5} \cos(2\varphi) \text{ [pu]}
\]

and the mutual inductance in

\[
L_{ca} = -0.832 \cdot 10^{-3} + 6.25 \cdot 10^{-5} \cos\left(2\left(\varphi - \frac{2\pi}{3}\right)\right) \text{ [pu]}
\]

Following this result it is clearly discernible that the assumption of an equal amplitude of self and mutual inductance of the expanded Park theory is incorrect. The amplitude of the mutual inductance is twice the self inductance. Even assumption of the steady component is inaccurate. The leakage inductance, calculated by a FE method described in [5], is \( L_\sigma = 1.85 \cdot 10^{-5} \text{ pu.} \)

(1.824 \cdot 10^{-3} - 1.85 \cdot 10^{-5}) \neq -2 \cdot -0.832 \cdot 10^{-3} \quad (16)

3.2 Salient pole rotor synchronous machine

The salient pole rotor machine shows the same results as the round rotor machine. The inductances are

\[
L_{aa} = 0.78 \cdot 10^{-3} + 1.2 \cdot 10^{-5} \cos(2\varphi) \text{ [pu]}
\]

\[
L_{ca} = -0.33 \cdot 10^{-3} + 2.97 \cdot 10^{-5} \cos\left(2\left(\varphi - \frac{2\pi}{3}\right)\right) \text{ [pu]}
\]

\[
L_\sigma = 9.26 \cdot 10^{-6} \text{ [pu]}
\]

According to the more unsymmetrical rotor geometry the amplitude of the inductances are higher than from the round rotor machine. But again the prediction of a same amplitude of stator self and mutual inductance is not fulfilled.
To investigate the influence of saturation to the inductances the round rotor synchronous machine is modeled with non linear magnetic material. The inductances are calculated at no load operation with rated voltage. At this operation point, the machine is highly saturated. The inductance waveforms are depicted in Fig. 6. Due to the high saturation of the direct axis all inductances have a phase shift of $\frac{\pi}{2}$ rad, a smaller steady component and a growing amplitude in comparison to the synchronous machine with linear material. However, the inductances reveal the same behavior even at other operation points. The values of the self and the mutual inductance are not equal. Again this shows the wrong assumption of the expanded Park theory.

### 3.4 New calculation of Park inductances

According to the results of chapters 3.1 to 3.3 the inductances of eq. (6) and (7) have to be restated:

$$L_{aa}(\varphi) = L_\sigma + M_1 + M_2 \cdot \cos(2\varphi \varphi)$$

$$L_{ca}(\varphi) = -\frac{M_1}{2} + M_2 \cdot \cos\left(2\varphi \left(\varphi - \frac{2\pi}{3}\right)\right)$$

with

$$M_1 \neq M_1'$$

$$M_2 \neq M_2'$$

These inductances are just valid without considering saturation. With the new stator self and mutual inductances the Park transformation does not lead to a decoupled system:

$$\begin{bmatrix}
L_{0}' & \frac{M_{dq}}{2} \cos(3\varphi) & -\frac{M_{dq}}{2} \sin(3\varphi) \\
-\frac{M_{dq}}{2} \cos(3\varphi) & L_d' & 0 \\
\frac{M_{dq}}{2} \sin(3\varphi) & 0 & L_q'
\end{bmatrix}$$

with

$$L_0' = L_\sigma + M_1 - M_1'$$

$$L_d' = L_\sigma + M_1 + M_2 + \frac{M_1' + M_2'}{2}$$

$$L_q' = L_\sigma + M_1 - M_2 + \frac{M_1' - M_2'}{2}$$

$$M_{dq} = M_2 - M_2'$$

### 4. Identification of Park Parameter

In chapter 3, it is shown that the expanded Park theory bases on wrong hypothesis. However, the Park equations are used as the standard analytic tool for calculating steady state operations and transients of the synchronous machines. Therefore there is high demand on Park parameters for each machine. But as shown the Park parameters should not be de-
rived of the geometric dimensions of the synchronous machines. The classical procedure of deriving the parameters is the measurement of the machine [6].

In this paper the parameters are determined with the evolution strategy by analyzing the three phase short circuit from no-load operation. First, the influence of the saturation is neglected. After the parameters are determined, the saturation can be regarded with different methods, e.g. described in [3].

In this paper the influence of initial parameters of the evolution strategy is under investigation. First, there are no restrictions to the parameter, they are chosen randomly in the domain. In a second investigation the initial parameters of the d-axis are predicted by the classical method described in [6]. Both investigations use the same settings.

The theory of the ES, especially of the creation of a new generation, is well known in literature, e.g. in [7]–[10].

4.2 Analyzed machine

In the following only the parameters of the round rotor synchronous machines with linear magnetic material are determined. It is a 50 kW synchronous machine with \( \cos \phi = 0.8 \).

The finite-element model of the inductances analysis is used to calculate the three phase short circuit currents and the occurring electrical torque. The resistance of the stator and the field winding can be measured easily. Hence, it is necessary to determine nine parameters.

4.3 Random start parameters

The domain of the parameters is set to:

\[
\begin{align*}
\{ R_x \in \mathbb{R} \mid -0.5 \, \text{pu} & \leq R_x \leq 0.5 \, \text{pu} \} \\
\{ L_x \in \mathbb{R} \mid -12 \, \text{pu} & \leq L_x \leq 12 \, \text{pu} \}
\end{align*}
\]

Due to the fact, that the parameters of Park’s equations cannot be determined from the phase-domain parameters, it is valid that the parameters can attain negative values. They are totally detached from physics.

4.1 Evolution strategy

The evolution strategy (ES) is a subset of the evolutionary algorithm that follows the natural biological evolution. The procedure of the ES can be separated into six steps:

1) Defining the fitness function
2) Initializing of the starting population
3) Evaluating the actual fitness value
4) Creation of a new generation
5) Deterministic selection
6) Repeat on steps 4) to 5) until termination

Fig. 7: Stator current at three phase short circuit

Fig. 8: Three phase short circuit: ES results with random start parameters (FEM green, Park red)
The initial parameters are stochastic randomly chosen in the whole domain. Creating the 1000th new generation stops the ES. The determined parameters are shown in the appendix. Fig.8 shows the comparison of the finite-element simulation and the analytic Park simulation results. It is obvious that this set of parameters is not applicable.

4.4 Predicted start parameters

Before the evolution strategy for determining the Park parameters starts, the initial parameters are predicted by evaluating the three phase short circuit. These method just leads to the parameters of the direct axis. Therefore the quadrature axis parameters are set equal to the d-axis parameter. The rotor inductances $L_{af}$ and $L'_{af}$ are functions of the Canay inductance $L_{cd}$ [11], but in [6] there is no method to determine this inductance. So first the inductance is set zero.

![Fig. 9: Three phase short circuit: ES results with predicted start parameters (FEM: ——— Park: ———)](image)

The settings of the ES are the same as the investigation with random parameters. In Fig. 9 it can be seen, that the optimized parameter set (see appendix) gives nearly perfect accordance to the finite element model. There is only a small deviation at the field current $I_f$.

### Table I: Predicted start parameters [pu] determined analytically

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<th>$R_q$</th>
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4.5 Two phase short circuit

The parameters are determined by an optimization of the three phase short circuit. They have to be proofed if they will be still correct with other operations of the synchronous machine. Therefore a two phase short circuit is calculated with Park’s equations and compared to a numerical calculation with FEM.

![Fig. 9: Two phase short circuit: ES results with predicted start parameters (FEM: ——— Park: ———)](image)

Again, the analytic model shows a good accordance to FEM values. So, the parameters fit for calculating the transient behavior of this synchronous machine.

5. Conclusion

In this paper it is shown, that the expanded Park theory is based of wrong assumptions. The inductances of two different types of synchronous machines are calculated with the FEM software Ansys Maxwell 2D. The simulation results show clearly, that the assumptions for the expanded Park transformation lead to an improper model. It is then reasonable to use the phase-domain model without any transformation. However, the Park theory is state of the art for analytic calculations of the synchronous machine. Therefore, a method is shown in this paper that determine the needed parameters. The method is based on the evolution strategy and uses the current and
torque values of a three phase short circuit. The values can be measured directly or calculated with the use of the finite element method. It is strongly recommend to predict the initial parameters. The two phase short circuit shows, that the determined parameters are also valid for other transients.

6. References


Appendix

Set of parameters I: Determined with random start parameters [pu]

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Set of parameters II: Determined with predicted start parameters [pu]

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