MINIMIZATION OF POWER LOSSES OF A WOUND ROTOR SYNCHRONOUS MOTOR FOR TRACTION DRIVES USING LOLIMOT APPROXIMATION OF THE MAGNETIZING CHARACTERISTICS

Abstract: Over the last years, there has been a rising interest in wound-rotor synchronous motors (WSM) for electric vehicle drive application. They provide an alternative to permanent magnet motors (PSM) which are subject to shortage issues with the rare-earth material supply [5]. Although an analytical solution of the WSM minimum copper losses problem exists [1], the associated assumptions of unlimited voltage and constant motor inductions are mostly not applicable for electric vehicle applications. This paper proposes a simple method to improve the convergence of a numerical solution of the copper loss minimization problem subject to a nonlinear equality constraint (motor torque) and inequality constraints (available voltage and maximum current). A set / tree of weighted locally linearized (LoLiMoT) models replaces nonlinear lookup tables specifying the inductances $L_d$, $L_q$, $L_f$ in terms of the currents $I_d$, $I_q$ and $I_f$. A benefit of this is that the analytical expressions for the Jacobian and the Hessian matrices of the objective function and the constraints functions are readily available to a gradient-based numerical optimization function. The calculation results presented in the paper show a reduction of the necessary number of iteration steps. In addition, straightforward modification of the presented method allows optimization also under consideration of magnetic losses.

1. Introduction

Power loss minimization is one of the most important and challenging goals in development of high efficiency electrical traction drives for electric vehicles. In case of a PSM, power loss minimization is achieved by choosing a pair of loss minimizing stator currents (direct ($I_d$) and quadrature ($I_q$) currents) for a given torque request and speed (mechanical operating point). In case of a WSM, the excitation current ($I_f$) adds an extra degree of freedom. This increases the complexity of the loss minimization problem, as the control unit is expected to generate an optimal current triplet (electrical operating point) $I_d$, $I_q$ and $I_f$ for every mechanical operating point of the machine in a way that guarantees a power losses minimum and thus an increase in the efficiency of the machine.

For a traction machine, high torque demand dynamics and efficiency are some of the top goals to achieve subject to varying system state. Due to nonlinear loss and system functions, the optimal steady-state currents are usually calculated offline using a time intensive process for relevant mechanical operating points under consideration of system state variation such as temperatures and voltages. Usually, a set of multi-dimensional look-up tables arises from this process.

An analytical solution of the WSM current command exists, but the imposed assumptions of an unlimited motor voltage demand and constant motor inductions ($L_d$, $L_q$ and $L_f$) do not typically hold for an electric traction machine, particularly in the case of WSM which exhibit significant nonlinearity of magnetizing currents.

2. Mathematical Model of a WSM

2.1. Electromagnetic Equations

The equations of a WSM are similar to those of a PSM. An additional electrical rotor circuit equation and a corresponding flux-linkage term describe the rotor flux and electrical quantities. For the stator voltages, the following equations in the rotating dq-coordinate system hold [1]:

$$U_q = R_d I_q + \frac{d\psi_q}{dt} + \omega_s \psi_d$$  \hspace{1cm} (1)

$$U_d = R_d I_d + \frac{d\psi_d}{dt} - \omega_s \psi_q$$  \hspace{1cm} (2)

where, $U_d$, $U_q$, $I_d$, $I_q$ are the components of the stator voltage and current, $\psi_d$, $\psi_q$ are the sta-
tor flux components, $R_s$ is the stator resistance and $\omega_s$ is the stator angular frequency.

For rotor voltage the following holds:

$$U_f = R_f I_f + \frac{d\psi_f}{dt}$$  \hspace{1cm} (3)

where $U_f, I_f$ are the excitation voltage and current, $\psi_f$ is the rotor flux and $R_f$ is the rotor resistance.

The following set of equations describes the flux linkage of a WSM:

$$\psi_a = L_a I_a + L_{ha} I_f$$  \hspace{1cm} (4)

$$\psi_q = L_q I_q$$  \hspace{1cm} (5)

$$\psi_f = L_f I_f + L_{hd} I_d$$  \hspace{1cm} (6)

where $L_{ha}$ is the main (coupling, mutual) inductance in the direction of the d-axis.

For electrical machine torque we can write:

$$T = \frac{3}{2} Z_p (\psi_a I_q - \psi_q I_a)$$  \hspace{1cm} (7)

where $Z_p$ is the number of pole pairs.

Clearly, infinitely many feasible current triples exist for every torque request with inactive inequality constraints. The corresponding power losses of every feasible electrical operating point is described by equation (8).

### 2.2. Power Losses

For the sake of simplicity, magnetic losses are neglected in this paper. Equation (8) defines the total electrical losses.

$$P_{cu} = \frac{3}{2} (I_a^2 + I_d^2) R_s + I_f^2 R_f$$  \hspace{1cm} (8)

Stator resistance increase with stator frequency due to skin effect is also neglected. This assumption holds because the maximum considered speed gradient is typically slow relative to the current control bandwidth and thus speed can be considered as a quasi-stationary parameter / input.

### 2.3. System Limits

A battery with limited, load dependent voltage supplies the electrical system of a vehicle. The available voltage can strongly vary as opposed to typical industrial applications. The DC-link voltage limits the maximum stator voltage as shown in equation (9) and the feasibility domain:

$$U_s = \sqrt{U_{a}^2 + U_{d}^2} \leq \frac{U_{dc}}{\sqrt{3}}$$  \hspace{1cm} (9)

The last formula considers only steady state of equations (1), (2) and (3), since the goal is to accelerate an offline steady state current command optimization problem.

In addition to the voltage inequality constraint as given by equation (9), the maximum stator current is also subject to an inequality constraint due to the current limits of inverter:

$$I_s = \sqrt{I_{a}^2 + I_{d}^2} \leq I_{s\text{\,max}}$$  \hspace{1cm} (10)

### 3. Optimization Problem Formulation

Using the system description and constraints introduced in the previous paragraph, one can formulate the optimization problem as follows:

$$\begin{align*}
\text{min}_{[\psi_a, I_a, I_d]} & \left( \frac{3}{2} I_a^2 R_s + I_f^2 R_f \right) \text{st.} \\
T - \frac{3}{2} Z_p (\psi_a I_q - \psi_q I_a) &= 0 \\
U_s^2 - \frac{U_{dc}}{\sqrt{3}} &\leq 0 \\
\sqrt{U_{a}^2 + U_{d}^2} - \frac{U_{dc}}{\sqrt{3}} &\leq 0 \\
\sqrt{I_{a}^2 + I_{d}^2} - I_{s\text{\,max}} &\leq 0
\end{align*}$$  \hspace{1cm} (11)\text{--}(14)

This set of equations represent a general constrained optimization problem with one equality constraint defined by the torque equation and two inequality constraints given by system limits on current and voltage.

A solution exists and is optimal if a feasible point fulfils Kuhn-Tucker conditions.

To find the optimal reference currents, the \textit{fmincon} function in Matlab is one possible option to solve the formulated problem numerically and is our choice in this work.

The approximation of the gradients for a gradient-based solvers leads to multiple evaluations of the perturbed system.

On the other hand, a LoLiMoT model structure, as shown below, provides a general analytical formula for Jacobi and Hessian matrices of the
constraints, which are used in order to sub-
sequently speed up the solution of the optimiza-
tion algorithm.

4. Local Linear Model Trees (LoLiMoT)
The LoLiMoT algorithm is a fast incrementa-
construction algorithm for local linear neuro-
fuzzy models also known as Takagi-Sugeno
fuzzy models described in [2] - [3].
The algorithm approximates a nonlinear func-
tion with a set of weighted locally linear models
as described in equations (15) – (18). The
weighting functions of the partial models (see
equation (18)) are Gaussian probability density
functions centered in the operating points of the
respective partial models. The model output \( y \)
also calculated by:

\[
y = \frac{F(x)}{H(x)}
\]  
(15)

where:

\[
F(x) = \sum_{i=1}^{N} \gamma_i(x) \Phi_i(x)
\]
(16)

\[
H(x) = \sum_{i=1}^{N} \Phi_i(x)
\]
(17)

where:

\[
\Phi_i(x) = e^{-\frac{1}{2} \sum_{j=1}^{M} \frac{(x_j - \xi_{ij})^2}{\sigma_{ij}^2}}
\]
(18)

5. Partial Derivatives of LoLiMoT Models
Since exponential function is analytic, the de-
scribed model structure clearly provides ana-
tical expressions for partial derivatives of the
approximated function with respect to input vari-
able.
Taking the first partial derivative of equation
(15) with respect to \( x_j \) gives:

\[
\frac{\partial y}{\partial x_j} = \frac{D_j(x)}{H(x)} - \frac{F(x) S_j(x)}{H(x)^2}
\]
(19)

and

\[
S_j = \frac{\partial H}{\partial x_j} = -\sum_{i=1}^{N} \Phi_i(x_j - \xi_{ij}) \sigma_{ij}^2
\]

Differentiating equation (19) with respect to \( x_j \)
yields:

\[
\frac{\partial^2 y}{\partial x_k \partial x_j} = \left( T_{jk} H - D_k S_j - D_j S_k + \frac{F}{H} \frac{\partial^2 S_j}{\partial x_k} - \frac{\partial S_j}{\partial x_k} \right)
\]
(20)

where

\[
T_{jk}(x) = \frac{\partial D_j(x)}{\partial x_k}
\]
and

\[
Z_{jk}(x) = \frac{\partial S_j(x)}{\partial x_k}.
\]

6. Jacobian and Hessian of the Objective
Function, Equality and Inequality con-
straints
The Jacobian matrix is defined as a matrix of
gradients of a multivariable scalar function \( f \),
and can be calculated for the objective function,
equality and inequality constraint functions by
its definition (21).

\[
I_f = \begin{bmatrix}
\frac{\partial f}{\partial l_a} & \frac{\partial f}{\partial l_q} & \frac{\partial f}{\partial l_r}
\end{bmatrix}
\]
(21)

For the Hessian matrix \( H_f \) of a scalar multi-
variable function, one can write:

\[
H_f = \begin{bmatrix}
\frac{\partial^2 f}{\partial l_a^2} & \frac{\partial^2 f}{\partial l_a \partial l_q} & \frac{\partial^2 f}{\partial l_a \partial l_r} \\
\frac{\partial^2 f}{\partial l_q \partial l_a} & \frac{\partial^2 f}{\partial l_q^2} & \frac{\partial^2 f}{\partial l_q \partial l_r} \\
\frac{\partial^2 f}{\partial l_r \partial l_a} & \frac{\partial^2 f}{\partial l_r \partial l_q} & \frac{\partial^2 f}{\partial l_r^2}
\end{bmatrix}
\]
(22)

7. Simulation
For simulation and testing, the following oper-
at- ing range and limits were chosen:

- Maximum DC-link voltage: 350 V
- Mechanical speed range: 0 - 15000 rpm
- Torque range: 0 - 300 Nm
The values of nonlinear machine model parameters resulted from a FEM computation in the design phase. In this paper, we take into account three approaches for handling the nonlinearities of the inductances in the current optimization problem. All simulations were performed with the same `fmincon` solver configuration; the algorithm was set interior-point.

**Approach 1**

This approach represents the baseline; the original, interpolated set of three dimensional flux lookup tables representing the nonlinear state dependence of the $L_d$, $L_q$, $L_f$ and $L_{hd}$ are used directly as an input for the optimization problem.

**Approach 2**

In this approach, the three dimensional flux lookup tables are simply replaced with a corresponding LoLiMoT approximation.

**Approach 3**

In this approach, we provide the analytical formula for the gradient and Hessian (19), (20), (21) and (22) to the solver.

### 8. Results

Figures 1, 2 and 3 show the iteration results of optimization problem (11) at a single operating point using the three aforementioned approaches. The results are shown for an operating point of $T = 11.5$ Nm and $n = 0$ rpm. The initial conditions given to the algorithm were the currents $[I_d, I_q] = [-10, 0, 0]$ A. The figures are 2D-representations visualizing the stator currents $I_d, I_q$ only. Tab. 1, 2 and 3 show the number of iterations and total run time for a randomly selected mechanical operating point as well as for all points within the operation range. The results show (see Tab. 1, 2, 3) a reduced number of iterations and run time for approach 3 compared to the reference approach 1, whereas approach 2 results in slightly increased number of iterations but with reduced algorithm run time. The optimal points and power losses found in simulations of approaches 2 and 3 slightly deviated from the reference simulation of approach 1. This results from the additional local error caused by the nonlinear function approximation using the LoLiMoT model. This fact has, in our opinion, no major impact on the presented results, since our aim purpose was an investigation of the convergence.

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Iterations</th>
<th>Run Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach 1</td>
<td>17</td>
<td>2.56</td>
</tr>
<tr>
<td>Approach 2</td>
<td>17</td>
<td>1.96</td>
</tr>
<tr>
<td>Approach 3</td>
<td>5</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Tab. 2. Number of iterations over all operating points

<table>
<thead>
<tr>
<th>Approach</th>
<th>Max. Iterations</th>
<th>Min. Iterations</th>
<th>Avg. Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach 1</td>
<td>17</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Approach 2</td>
<td>22</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Approach 3</td>
<td>13</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Tab. 3. Runtime over all operating points

<table>
<thead>
<tr>
<th>Approach</th>
<th>Total Run Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach 1</td>
<td>1250.4</td>
</tr>
<tr>
<td>Approach 2</td>
<td>546.3</td>
</tr>
<tr>
<td>Approach 3</td>
<td>145.6</td>
</tr>
</tbody>
</table>

9. Conclusion

In this paper we showed that the number of steps needed to reach an optimal operating point for a given torque request can be reduced if the Jacobi and Hesse matrices can be expressed analytically. The reduction of the run time is lower relative to the reduction of the number of iterations. A possible explanation is the higher computational complexity of the exponential function evaluations. Through the reduction of computation effort, the presented approach represents a promising preliminary step in our attempt to formulate the motor current calculation problem in a form suitable for a real-time, online implementation of operating point optimization with guaranteed convergence and suboptimality bounds.

Bibliography


Authors

Dr. Viktor Barinberg, IAV GmbH, Weimarer Straße 10, 80807 München, DE, viktor.barinberg@iav.de
Petr Micek, IAV GmbH, Weimarer Straße 10, 80807 München, DE, petr.micek@iav.de
Seif Shaaban, IAV GmbH, Weimarer Straße 10, 80807 München, DE, seif.eldin.tarek.shaaban@iav.de