NONLINEAR MODEL OF A SYNCHRONOUS GENERATOR FOR ANALYSIS OF MORE ELECTRIC AIRCRAFT POWER SYSTEMS

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Abstract: A nonlinear model for studying a variable-speed synchronous generator (SG) in more electric aircraft (MEA) power system has been developed. The saturation effects of the SG magnetic circuit have been considered. The model has been implemented in the Synopys/Saber simulation environment. The modelling language MAST has been used to elaborate the SG model. The model exhibit a network with the same number of external terminals/ports as the real SG, and represents its behaviour in terms of the electrical (stator and rotor windings) and mechanical (shaft) variables as well. The main advantage of the approach is the ease of describing MEA power system in terms of its topology. Thus, normal and fault operation of any MEA power system can be effectively investigated. The presented simulation results have proved that the proposed approach can be recommended for analysis of MEA power systems.

1. Introduction

A lot of efforts are currently done into development on the concept of the more electric aircraft (MEA) for the electrical design system of commercial aircraft, usable by the business and regional aircraft and rotorcraft as well. The fundamental MEA concept, which removes hydraulic, pneumatic and gearbox driven subsystems in favour of electrical driven subsystems, has necessitated the development of high performance starter/generator systems and compact lightweight electric drives and servo subsystems. As a consequence high power synchronous generators (SG) are required to supply the increased electrical power needs on board [4, 5, 9, 11, 12].

In MEA power systems SG are driven directly by variable speed prime movers (jet engines), and therefore, operate as variable frequency machines. Compared with the conventional constant frequency generator (50/60 Hz for utility grid and 400 Hz for aircraft systems), the GCU for the variable frequency system must deal with significant variations in operating points and associated dynamics due to a wide frequency range (greater than 2:1 in some MEA systems).

A typical electric power generator in MEA applications is actually composed of two synchronous machines, a rectifier and a small permanent magnet generator (PMG), on the same shaft [5, 10]. One synchronous machine is the main generator and the other smaller lower rating machine with its field winding on the stator works as the brushless exciter.

For the evaluation of aircraft on-board electric power systems and electric servo systems with regard to their weight, behaviour and reliability a novel modelling and simulation tools are being developed [2]. One essential requirement for a simulation environment of MEA power systems consisting of many nonlinear components is mulitphysical modelling that allows to simulate all aircraft systems, which use different forms of power, in one integrated model. Different physical domains have to be considered in the simulation of complex aircraft systems with high computational efficiency. The integrated model uses component models, that are being delivered by the equipment manufacturers, to compose an integrated aircraft systems model.

A key technique in achieving this is the use of an advanced network solver such as Synopsys/Saber based on a mixed-signal hardware description language called MAST [13, 14]. While using the modelling language MAST you are not only able to develop the various mathematical-based models you need, but are also able to develop mixed-signal and multi-physical (mixed-technology) models. Moreover, MAST models can be made at any level of abstraction – from simple transfer function descriptions, to
detailed physics-based descriptions. And they can be mixed throughout multiple levels of hierarchy.

The importance of SG in MEA power systems has been well recognized. They are highly nonlinear, complex electromechanical device, whose dynamic behaviour directly impacts the performance and reliability of the power system network [3, 5, 10, 11, 12]. To analyze the dynamic behaviour of the SG, an effective and accurate simulation model is desired. However, it is difficult to develop such a model due to the nonlinear inductances of the SG windings. Further, the model needs to account for dynamics involving electrical and mechanical domains.

In the paper [9] a background has been discussed for using the MAST language to model a SG for investigating the multi-physical power behaviour of MEA power systems. However, the magnetic circuit saturation effects have not been considered.

This paper describes the use of the Synopsys/Saber and MAST language for modelling the SG magnetic saturation in both the \( q \) and \( d \)-axis while keeping the terminal characteristics of the machine in the physical domain. This approach simplifies the interconnection of the machine model with power electronic rectifiers/converters. It has been shown that this SG model is computationally efficient and suitable for dynamic time-domain MEA power system studies.

### 1. SG modelling

#### 1.1. Hypothesis and assumptions

A SG, according to its degree of freedom, can be represented as a multi-port electromechanical converter with pair of terminals (ports), which are the windings and shaft terminals (ports).

The structure of a SG model depends upon the assumed modelling of the energy transformation, energy conversion, energy accumulation and energy dissipation processes in SG.

For developing the SG model in terms of its ports/terminals variables, i.e., especially for MEA power system analysis and design, the general equations of motion of SG are recalled, and next a combined variables transformations from the natural stator as \( hs cs cs \) axes to \( q d 0 \) axes, and vice versa, are performed under the conventional assumptions [1, 3, 6, 7, 8, 9, 10, 11].

#### 1.2. Model equations and its implementations into MAST language

Voltages, currents and flux-linkages equations in terms of \( q d 0 \) axes variables:

\[
v_{qs} = r_s i_{qs} + \omega_s \psi_{ds} + \frac{d\psi_{qs}}{dt} \tag{1}
\]

\[
v_{ds} = r_s i_{ds} - \omega_s \psi_{qs} + \frac{d\psi_{ds}}{dt} \tag{2}
\]

\[
v_{ds} = r_s i_{ds} + \frac{d\psi_{ds}}{dt} \tag{3}
\]

\[
0 = r_s i_{ds} + \frac{d\psi_{ds}}{dt} \tag{4}
\]

\[
v_{fd} = r_d i_{fd} + \frac{d\psi_{fd}}{dt} \tag{5}
\]

\[
0 = r_d i_{fd} + \frac{d\psi_{fd}}{dt} \tag{6}
\]

where, \( \omega_s = \rho \omega_m ; \rho \) - number of pole pairs, \( \omega_m \) - angular rotor (mechanical) speed.

The above equations in terms of MAST language:

\[
iqs: v_{qs} = R_s i_{qs} + w_e \psi_{ds} + \frac{d\psi_{qs}}{dt} \tag{7}
\]

\[
ids: v_{ds} = R_s i_{ds} - w_e \psi_{qs} + \frac{d\psi_{ds}}{dt} \tag{8}
\]

\[
i0s: v_{0s} = R_s i_{0s} + \frac{d\psi_{0s}}{dt} \tag{9}
\]

\[
i0s: v_{0s} = R_s i_{0s} + \frac{d\psi_{0s}}{dt} \tag{10}
\]

\[
\text{torque and angular speed equations:}
\]

\[
T_m + T_e = J \frac{d\omega_m}{dt} + B_m \omega_m \tag{11}
\]

\[
\omega_m = \frac{d\alpha}{dt} \tag{12}
\]

\[
T_e = \frac{3}{2} \rho (\psi_{ds, i_{qs}} - \psi_{qs, i_{ds}}) \tag{13}
\]

The above equations in terms of MAST language:

\[
\text{torque equation}
\]

\[
\text{visc} = B_m \times \omega_m
\]

\[
T_e = \frac{3}{2} \rho (\psi_{ds, i_{qs}} - \psi_{qs, i_{ds}})
\]

\[
\text{nom} = d_{by_dt}(\text{nom})
\]

\[
\alpha = \text{wmd}_{by_dt}(\alpha)
\]

### 1.3. Modelling magnetic saturation effects

Most of the methods currently being used to calculate the saturated values of inductances are based on the open circuit saturation curve for magnetizing inductances, and on the short-circuit saturation curve for leakage inductances.
as a reference to find the saturation-dependent inductances in functional forms [1, 4, 7, 8, 11]. In general, for a given saturation curve \( \psi = \psi(i) \), the inductances can be expressed as follows:

\[ L = \frac{\psi}{i} \quad \text{and} \quad \bar{L} = \frac{\partial \psi}{\partial i}. \]

Where, \( L \) is referred to as static (chord slope) saturated inductance, and \( \bar{L} \) as dynamic (tangent-slope or incremental) inductance. It should be noticed that when the flux linkage \( \psi \) is assumed as a state variable then the dynamic inductance is not necessary.

The model of saturation effects can be further developed by assuming a fictitious equivalent coil (i.e. the magnetic axis of the fictitious equivalent coil coincides with the resultant MMF due to the total current \( i \)) which carries a current \( i \) producing a flux-linkage \( \psi \).

It should be noted that the approach making use of the unique magnetization characteristic can be applied directly to smooth-air-gap machine. In the salient-pole machine, because of the physical saliency, in general, the magnetizing flux-linkage space vector is not coaxial with the magnetizing current space vector (the resultant flux wave is not collinear with the resultant MMF wave). Thus it is more problematic to characterise the saturation level in the saturated salient-pole machine than in the saturated smooth-air-gap machine, where the magnetizing flux-linkage space vector is coaxial with the magnetizing-current vector and where the amplitude of the magnetizing flux-linkage space vector is a non-linear function of the magnetizing current. However, it may be assumed that the pole tips and/or the teeth are the most saturated parts of the magnetic circuit, and therefore, as for the smooth-air-gap machine, in each axis the saturation level is determined by the amplitude of the magnetizing-current space vector.

According to the above consideration the magnetic saturation of the SG magnetic circuit can be represented by the following characteristic:

\[ \psi_m = f(i_m) \]

(10)

where, the total magnetizing current \( i_m \) is referred to the turns of field winding, and is defined as following:

\[ i_m = \sqrt{i_{md}^2 + i_{mq}^2} \]

(11)

\[ i_{md} = \frac{1}{k_{fd}} (i_{fs} + i_{ds} + i_{kd}) \]

(12)

\[ i_{mq} = \frac{1}{k_{fd}} (i_{qs} + i_{kq}) \]

(13)

where, coefficient \( k_{fd} = \frac{L_{sfd}}{L_{md}} \).

The coefficient \( k_{fd} \) refers the mutual stator–field inductance \( L_{sfd} \) to the magnetizing inductance \( L_{md} \).

To represent the characteristics \( \psi_m = f(i_m) \) for the \( q \) and \( d \) axes flux-linkages, in terms of the SG inductances, the following functions are used:

\[ L_{md\_sat} = k_{sat} (i_m) L_{md} \]

\[ L_{mq\_sat} = k_{sat} (i_m) L_{mq} \]

(14)

where, \( k_{sat} \) – saturation coefficient, \( L_{md\_sat} \) – saturated magnetizing inductances in \( d \) and \( q \) axis respectively; \( L_{md} \), \( L_{mq} \) – unsaturated magnetizing inductances in \( d \) and \( q \) axis respectively. It has been assumed that the coefficient \( k_{sat} \) has the same value for the magnetic circuit in \( q \) and \( d \) axes, respectively.

Using the above relations the SG flux linkages, used in the equations (1)–(6) and (9), are defined as following:

\[ \psi_{qs} = L_{ls} i_{qs} + k_{sat} L_{mq} (i_{qs} + i_{kq}) \]

(15)

\[ \psi_{ds} = L_{ls} i_{ds} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd}) + \psi_{rem} \]

(16)

\[ \psi_{0s} = L_{ls} i_{0s} \]

(17)

\[ \psi_{kq} = L_{lkq} i_{kq} + k_{sat} L_{mq} (i_{qs} + i_{kq}) \]

(18)

\[ \psi_{fd} = L_{lfq} i_{fd} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd}) \]

(19)

\[ \psi_{kd} = L_{lfq} i_{kd} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd}) \]

(20)

where, \( \psi_{rem} \) – remanence flux.

The saturation coefficient \( k_{sat} \) is evaluated using the no-load voltage characteristic. This characteristic is measured as following: open circuit terminal voltage \( V_{q0} \) versus field current \( I \) (actual field current), and open circuit terminal voltage \( V_{q0\_rem} \) versus remanence flux \( \psi_{rem} \).

Using the equations (1) and (16) for the no-load case of the SG, we have:

\[ V_{q0} = \omega \psi_{ds} \]

(21)
Using the equations (21) and (22) the saturated magnetizing inductance \( L_{\text{md sat}} \) and saturation coefficient \( k_{\text{sat}} \) can be calculated as following:

\[
L_{\text{md sat}} = \frac{\psi_{ds} - \psi_{rem}}{I_r} = \frac{V_q^0 - V_{q0rem}}{\omega I_r}, \tag{23}
\]

\[
k_{\text{sat}} = \frac{L_{\text{md sat}}}{L_{\text{sp fd}}}, \tag{24}
\]

where, the value of the mutual inductance \( L_{\text{sp fd}} \) is evaluated as \( L_{\text{sp fd}} = \max(L_{\text{md sat}}) \)

To implement the no-load voltage characteristic (saturation characteristic) of SG into the MAST language the table look-up tool (TLU) of the MAST language is used as a foreign routine. The following assumptions are used for this approximations: (i) linear interpolation, (ii) linear extrapolation at low and high ends of the no-load characteristic, respectively.

The data file containing the points of the no-load characteristic is elaborated as follows:

<table>
<thead>
<tr>
<th>field current (A)</th>
<th>no-load voltage RMS (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>0.30017</td>
<td></td>
</tr>
<tr>
<td>0.40023</td>
<td></td>
</tr>
<tr>
<td>0.57529</td>
<td></td>
</tr>
</tbody>
</table>

In the parameter section of the SG MAST model the following data preprocessing have been done:

```plaintext
# TLU data pre-processing (performs data checking, sorting and grid filling.)
data=tlu(0,1,sat_file,",",sat_interp,sat_extrap)
# TLU returns the sample points array for the independent variable
sp1=tlu(1,addr(data),1,sat_density) Sample Point Array
```

In the values section the TLU function is called to return the output value of the function for the given input values as following:

```plaintext
Vq0=tlu(2,addr(datap),1)
```

and

```plaintext
Lmdsat=sqrt(2)*vq0/(im*sat_rpm*math.pi*p/30)
phidf_rem=sqrt(2)*v_rem/(sat_rpm*math.pi*p/30)
Ksat=Lmdsat/Lspfd # saturation coefficient
```

The flux linkages equations (15)-(20) of the SG in terms of MAST language are following:

\[
\phi_{qs} = L_{ls qs} i_{qs} + k_{sat} L_{mq} (i_{qs} + i_{kq})
\]

\[
\phi_{ds} = L_{ls ds} i_{ds} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd}) + \phi_{fd rem}
\]

\[
\phi_{0s} = L_{ls 0s}
\]

\[
\phi_{kq} = L_{lkq} i_{kq} + k_{sat} L_{mq} (i_{qs} + i_{kq})
\]

\[
\phi_{fd} = L_{lfd} i_{fd} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd})
\]

\[
\phi_{kd} = L_{lkd} i_{kd} + k_{sat} L_{md} (i_{ds} + i_{fd} + i_{kd})
\]

2. SG simulations using Synopsys/Saber

To verify the proposed SG modelling approach, taking into account the saturation of magnetic circuit, the no-load and short circuit tests have been simulated. The tests have been carried out for a SG of the type GCe64o/1 produced by ELMOR.

2.1. No-load test

The topology of the simulated SG circuits is shown in Fig. 1, where the machine model is denoted by “ref:SME2009”. The “inside” of this model contain the equations and formulae presented in section 2. The results of the no-load test are shown in Fig. 2 and Fig. 3.

![Fig. 1. Topology of the simulated SG circuits for no load test](image)

2.2. 3-phase short circuit test

The topology of the simulated SG circuit is shown in Fig. 4, where the machine model is denoted by “ref:SME2009”. The “inside” of this model contain the equations and formulae presented in section 2. The results of the short circuit test are shown in Fig. 5 and Fig. 6. The constant angular speed operation of the SG has been assumed.

The simulation results have shown that magnetic saturation has an influence on the tested SG current values. The difference for the armature currents is about 6%. It has to be noticed that this relatively small difference refers to the
short circuit test, when the saturation level is low due to the armature demagnetization. The difference will be much larger for the SG transient states under loaded conditions. The comparison of the simulation and measured results for the studied SG will be presented at the conference.

Fig. 2. Open circuit voltage versus field current: measured (+) and simulation (-) results

Fig. 3. Coefficient $k_{sat}$, saturated inductance $L_{mdsat}$, and open circuit voltage versus field current: measured (+) and simulation (-) results

Fig. 4. Topology of the simulated SG circuits for short circuit test

Fig. 5. Simulation results of the short circuit test ($i_r$ – field current, $i_{as}$ – armature current – at the time of the short circuit the armature voltage has maximum value: linear (red line) and nonlinear (green line) magnetic circuit

Fig. 6. Simulation results of the short circuit test ($i_r$ – field current, $i_{as}$ – armature current – at the time of the short circuit the armature voltage has zero value: linear (red line) and nonlinear (green line) magnetic circuit

3. Conclusions

A nonlinear model for studying a variable-speed synchronous generator (SG) in more electric aircraft (MEA) power system has been developed. The saturation effects of the SG magnetic circuit have been considered. The model has been implemented in the Synopsys/Saber simulation environment. The modelling language MAST has been used to elaborate the SG model. The model exhibit a network with the same number of external terminals/ports as the real SG, and represents its behaviour in terms of the electrical (stator and rotor windings) and mechanical (shaft) variables as well. The main advantage of the approach is the ease of describing MEA power system in terms of its topology. Thus, normal and fault operation of any
MEA power system can be effectively investigated. The presented simulation results have proved that the proposed approach can be recommended for analysis of MEA power systems.

4. Bibliography


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