MODELLING OF A SEPARATELY EXCITED DC MOTOR SUPPLIED BY A THREE-PHASE BRIDGE RECTIFIER USING A COMPLEX FOURIER SERIES

Abstract: The presented paper deals with mathematical algorithms of an analytical calculus of an electromagnetic torque ripple of the separately excited DC motor. Assume the motor is supplied by a full controlled three-phase thyristors rectifier. To describe non harmonic rectifier output voltage, the complex Fourier series was used. Suppose the motor works with constant rotor speed at steady-state. Two motor operating conditions are discussed. Continuous and discontinuous armature current operations are distinguished. The waveforms of the motor current and electromagnetic torque are calculated.

1. Introduction
Modern power converters constitute the power stage for variable-speed dc drives. These power converters are chosen for a particular application depending on the number of factors such as cost, input power source, harmonics, power factor, noise and speed of response. Full controlled bridge rectifiers fed from three phase ac supply are considered in this paper.

2. Three-phase full controlled rectifier
A three-phase bridge thyristors full controlled rectifier is shown in Fig.1.

First of the voltage sequences can be expressed by a complex Fourier series of the form [3], [4]:

\[ u = \sqrt{3} \sqrt{2} U \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( A_k e^{j\beta k} + B_k e^{-j\beta k} \right) \right] \]  (1)

The Fourier coefficients take a form:

\[ a_0 = \frac{1}{\pi} (\cos \alpha - \cos \beta) \]
\[ A_k = \frac{1}{4\pi} (\sin^2 \beta - \sin^2 \alpha) - \]
\[ j \frac{1}{4\pi} \left( \beta - \alpha - \frac{1}{2} (\sin 2\beta - \sin 2\alpha) \right) \]

At a given instant, two thyristors are conducting. Assuming, that the voltage between phases \( a \) and \( b \) is maximum, then the thyristors \( T_1 \) and \( T_6 \) are conducting. At the next line voltage to get more positive than \( ab \) is \( ac \). At the time, the triggering signal for \( T_6 \) will be disabled and that of \( T_2 \) will be enabled. Similarly, it could be seen that the firing gating sequence is \( T_1, T_2, T_3, T_4, T_5, T_6 \) and so on. Also, each of these gating signals is spaced by a sixty electrical degree. [1], [2], [5]
The angles $\alpha$ and $\beta$ are defined:

$\alpha = 60^\circ + \alpha_r$;  $\beta = 60^\circ + \beta_r$;

$\alpha_r$ is the firing angle

$\beta_r$ is the extinction angle of the thyristors.

Other sequences are shifted by $\pi / 3$.

Other sequences are expressed:

$2u = \sqrt{6}U \left[ \frac{a_0}{2} + \sum_{k=1}^{6} (c_1 A_k e^{jk\omega t} + c^{-1}_1 B_k e^{-jk\omega t}) \right]$

$3u = \sqrt{6}U \left[ \frac{a_0}{2} + \sum_{k=1}^{6} (c^2_1 A_k e^{jk\omega t} + c^{-2}_1 B_k e^{-jk\omega t}) \right]$

$4u = \sqrt{6}U \left[ \frac{a_0}{2} + \sum_{k=1}^{6} (c^3_1 A_k e^{jk\omega t} + c^{-3}_1 B_k e^{-jk\omega t}) \right]$

$5u = \sqrt{6}U \left[ \frac{a_0}{2} + \sum_{k=1}^{6} (c^4_1 A_k e^{jk\omega t} + c^{-4}_1 B_k e^{-jk\omega t}) \right]$

$6u = \sqrt{6}U \left[ \frac{a_0}{2} + \sum_{k=1}^{6} (c^5_1 A_k e^{jk\omega t} + c^{-5}_1 B_k e^{-jk\omega t}) \right]$

The shifting coefficient $c_1 = e^{-jk\omega t/3}$

The three phase bridge rectifier output voltage is given by a sum of sequence voltages:

$u = \sum_{i=1}^{6} i u$

(3)

By substituting (2) and (3) into (1), the rectifier output voltage is expressed as:

$u = \sqrt{6}U \left[ 3a_0 + \sum_{k=1}^{6} (C_k A_k e^{jk\omega t} + C^{-1}_k B_k e^{-jk\omega t}) \right]$

(4)

The shifting coefficients are defined:

$C_k = 1 + c_1^k + c_2^k + c_3^k + c_4^k + c_5^k$

$C^{-1}_k = 1 + c^{-1}_1 + c^{-2}_1 + c^{-3}_1 + c^{-4}_1 + c^{-5}_1$

Fig.2 and Fig.3 shows the output voltage waveform of the three phase bridge rectifier in continuous and discontinuous output current zone.

3. Separately excited DC motor model

If the field winding is physically and electrically separate from the armature winding, then the machine is known as a separately excited dc machine. The independent control of field current and armature current endows simple but high performance control on this machine. The torque and the flux can be independently and precisely controlled. [2]

The equivalent circuit of a dc motor armature is based on the fact, that the armature winding has a resistance $R_a$, a self-inductance $L_a$, and an induced emf $U_a$. This is shown in Fig.1. The terminal relationship is written as

$u = R_a i_a + L_a \frac{di_a}{dt} + U_a$

(5)

If the field flux is constant, then the induced emf is proportional to the rotor speed $\omega_m$. The constant of proportionality is known as the induced emf constant. Then emf is represented as

$U_a = K_a \omega_m$

(5)

The air gap power is expressed in terms of the electromagnetic torque and speed

$P_e = M \omega_m = U_a i_a$

(6)

By substituting for the induced emf from (5), the electromagnetic torque is expressed as

$M = K_a \omega_m$

(7)
The voltage equation (4) is valid only in sequences of thyristors conducting. Consequently the induced emf cannot by assumed constant in the discontinuous current zone.

0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016
0 5 10 15 20 25 30 35 40 45 50
ui (V) t (s)

Fig.4. Induced emf waveform in discontinuous current zone

The induced emf can be for continuous and discontinuous zone expressed as a Fourier series of the form

\[
u_u = U_a \left[ 3a_{0a} + \sum_{k=1}^{\infty} \left( C_k A_k e^{jk\alpha t} + C_k^{-1} B_k e^{-jk\alpha t} \right) \right]
\]

The Fourier coefficients take a form

\[a_{0a} = \frac{1}{\pi} (\beta - \alpha)\]

\[A_{ka} = \frac{\sin k \beta - \sin k \alpha}{2k\pi} - j \frac{\cos k \alpha - \cos k \beta}{2k\pi}\]

\[B_{ka} = \frac{\sin k \beta - \sin k \alpha}{2k\pi} + j \frac{\cos k \alpha - \cos k \beta}{2k\pi}\]

In the Fig.4 is shown the waveform of the induced emf for the discontinuous current zone.

4. Motor current and torque calculation

The motor current can be calculated on the base of equation (4), (5) and (9). Equation (5) takes a form [6], [7]

\[
i_a = \sqrt{6}U_a \left[ 3a_{0a} + \sum_{k=1}^{\infty} \left( \frac{C_k A_k R_e}{R_e + j\omega L_e} e^{jk\omega t} + \frac{C_k^{-1} B_k R_e}{R_e - j\omega L_e} e^{-jk\omega t} \right) \right]
\]

The induced electromagnetic torque of the motor is calculated on the base of equation (8). Assume separately excited dc motor with the following parameters:

- \(P_h = 1.8kW\)
- \(U_{st} = 220V\)
- \(I_{st} = 9.6A\)
- \(n_x\) = 1300 rev/min
- \(R_e = 2.624\Omega\)
- \(I_f = 0.71A\)
- \(L_e = 38mH\)
- \(K_s = 1.3V/\text{rad}^{-1}\)

Three phase supply voltage of the rectifier:

- \(U = 3x180V/50Hz\) (Phase to phase voltage)

In the Fig.5 are shown the calculated waveforms of the dc supply voltage, motor current and torque at full rectifier output voltage \((\alpha_r = 12^\circ)\). The motor was loaded by a torque of \(T_p = 13.4Nm\) and its speed was 1300 rev/min.

\[
i_a = \sqrt{6}U_a \left[ 3a_{0a} + \sum_{k=1}^{\infty} \left( \frac{C_k A_k R_e}{R_e + j\omega L_e} e^{jk\omega t} \right) \right]
\]

Solving differential equation (10), the waveform of the motor current take a form

\[
i_a = \sqrt{6}U_a \left[ 3a_{0a} + \sum_{k=1}^{\infty} \left( \frac{C_k A_k R_e}{R_e + j\omega L_e} e^{jk\omega t} \right) \right]
\]
In the Fig.6 are shown the measured voltage and current waveforms. The commutation effect of the thyristors is strongly seen. Fig.7 and Fig.8 show the calculated and measured motor quantities rectifier firing angle of $\alpha_T = 18^\circ$. Motor was loaded by a torque of $M_p = 9.8\text{Nm}$ at the speed of 1300rev/min.

Fig.7. Calculated motor quantities

In the Fig.9 and Fig.10 show the state, when the speed decreases at 1050rev/min and firing angle at 39° by a constant load of $M_p = 9.8\text{Nm}$.

Fig.9. Calculated motor quantities

Fig.10. Measured motor quantities

Fig.11. Calculated motor quantities

Fig.12. Measured motor quantities

Fig.13. Calculated motor quantities
The Fig.11 and Fig.12 show the waveforms when the motor run at speed $1050\text{rev/min}$ and its torque is $2.4\,N\text{m}$. The firing angle was $41.5^\circ$. As seen from the figures, converter operating at the limit of the discontinuous currents.

![Fig.14. Measured motor quantities](image)

**Fig.14. Measured motor quantities**

Fig.13 and Fig.14 show the state, when motor is loaded by a very small load of $0.4\,N\text{m}$. The speed remained unchanged and was $1050\text{rev/min}$. The firing angle of the converter was $63^\circ$. As seen from the figures, converter operates with discontinuous currents.

**5. Conclusion**

In the paper is presented an analytical method using complex Fourier series enable to determine the armature current and torque waveforms of a separately excited dc motor. The method is valid for both continuous and discontinuous armature current zone. From the current waveforms, the induced electromagnetic torque ripple at steady state was determined. The calculus shows, that maximum torque ripple is in zone of small load and at low motor speed. Comparison of calculated and measured current and voltage waveforms is obvious enough accuracy of the calculus.

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**7. Bibliography**


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