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COMPARISON OF FEM AND LUMPED PARAMETER MODELS IN APPLICATION TO OPTIMIZATION OF A LSPMSM CONSTRUCTION

Abstract: The paper presents results of a comparison of two analysis methods of a Line Start Permanent Magnet Synchronous Motor (LSPMSM), which can be applied to optimization of its construction. Such important procedure properties, as computation time and the possibility of employing it in an automated process of searching for the optimal design were of particular interest. A supposition, that the time required for accurate FEM calculations and further processing of computation data can be a substantial constraint in a multi-parameter, multi-criterial optimization process, has been investigated. In the paper an attempt is made: a) to estimate how inaccuracy in prototype construction and uncertainty of material properties influence machine performances and b) what is a credibility of experimental verification of optimization computations. Such quantities as properties of permanent magnet and electromagnetic sheet, as well as a tolerance of some motor dimensions, have been taken in the analysis into account. The analysis results concern a few important machine properties: electromagnetic torque, stator current and rated efficiency. A selected machine optimization task is employed in the analysis.

1. Introduction

Growing interest in the field of energy saving and highly efficient devices directs more and more attention to the matter of designing line start permanent magnet synchronous motors (LSPMSM) and optimizing their construction [1]-[5]. Due to the complicated structure of the motor, designing is often aided by software based on finite elements method (FEM). The paper presents an attempt to compare two analysis methods, based on FEM calculations, and analytical model, developed and used by authors for optimization of construction [6],[7]. In the professional literature one can find many papers, in which FEM computations are verified by measurements on a prototype [1]-[3],[5],[8]. In this paper a related topic of credibility of measurement verification of applied models has been discussed, with an emphasis on a possible discrepancy between the design and existing motor dimensions and material physical properties. The usefulness of both models to design optimization is analyzed as well.

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2. Analytical model

Magnetically linear lumped-parameter model can be defined by means of linear equations set in the *dq* coordinate system. Parameters of this model are described by relationships, which represent machine properties, e.g. geometrical dimensions and physical properties of materials it was made of, see [9]. Such a model allows for relatively easy and fast modifications of a design, as well as extensive analysis of motor characteristics sensitivity to variation of selected parameters.

The developed model comprises procedures of determining the rated operation point, calculating the breakdown E-M torque and efficiency with its losses components, as well as the self-starting ability of the motor. In relation to [7], the model has been extended with heating calculations. They are similar to those ones used for a long time when designing optimal induction motors [10].

As was shown in [7], the proposed analytical model appeared to be an effective tool, which can be used when designing optimal high-efficiency LSPMSM. More detailed description of the model can be found in earlier author's papers [6],[11],[12].

3. FEM model

A description of the LSPMSM operation appears to be complex due to an interaction of two sources of the magnetic field. The first one is constant and is originated by permanent magnets, and the other, alternating one, comes from the ampere-turns of the stator. It is a reason that the FEM models are commonly employed to predict motor operation properties in the designing. In this paper, FEM models usefulness is analyzed. Such models have been used, which appeared to be helpful when verifying lumped-parameter models in the earlier authors papers [6],[7],[9],[11],[12].

One of the main obstacles to apply FEM models commonly is time of computations. It strongly depends on space discretization and the number of mesh elements. The choice of the mesh affects the accuracy of computations hence it is justified to differentiate it in specific parts of the machine, such as the air-gap, stator joke, shaft, rotor joke. Accepting its high density only in machine regions with high values of the magnetic energy gradient, it is possible to reduce the time of computations at a satisfactory accuracy level.

Two variants of the mesh discretization for the initial motor geometry have been shown on Fig. 1. They correspond with the highest and lowest number of nodes among cases presented on Fig. 2. All cases concern an operation point near the rated one. Despite the fact that a geometry representation for the lowest number of nodes seems to be questionable, it doesn't affect much the computation errors presented on Fig. 2. Moreover, such modification has reduced a dozen times the time of computations (from about 90 to about 5 minutes).

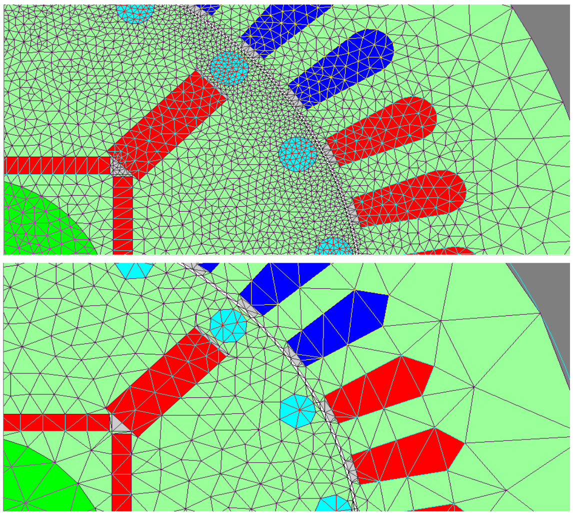

Fig.1 Different options of the mesh discretization in the FEM model.

As long as in the case of single computations given errors wouldn't cause much concerns from the engineering point of view, still in application to optimization situation changes. As the motor geometry varies in optimization, the number of mesh elements varies too. It results in a discontinuous change of motor quantities used for definition of the criterial functions and constraints. It eliminates application of gradient methods, as far as a problem-oriented procedure is developed for this particular situation. Available commercial software dedicated to FEM computations doesn't possess that property. Another unpleasant situation appears if equality constraints are used in optimization problem definition. In general, it is not possible to precisely meet these constraints for the same discontinuity reason, and it is a property independent on the optimization method selection. Such constraint occurs in the paper. One should also note that above-mentioned lack of accuracy have a cumulative character influencing the optimization routine. As a solution, a design can be selected, in which the minimum of objective function depends on the chosen discretization mesh more than on the construction properties.

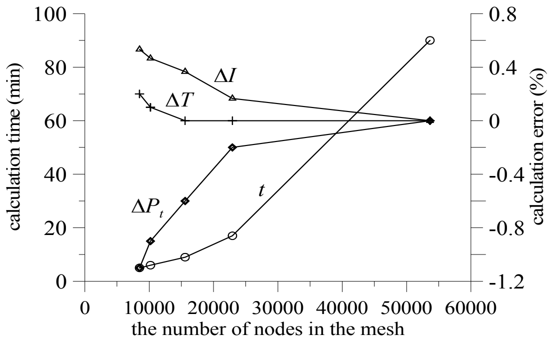

Fig.2 A relationship between computation time t and percentage errors ΔP_t (iron losses), ΔI (stator current), ΔT (electromagnetic torque), and the number of mesh nodes.

As in most cases, it is a good idea to keep common sense while searching for a compromise regarding the choice of mesh density. Results on Fig. 2 point out, that such a choice might be the case characterized by nodes number of about 20000 and errors in the range of 0.2%, with time of computations about 20 min.

The total time of FEM computations comprises a post-processing time as well. Each steady-state operation point of the LSPMSM is a result of transient calculations, due to an interaction between a stationary field of permanent magnets with alternating field originated by stator windings currents. For each computed case one should obtain fixed values, and next draw characteristics, define their range of computation and define coefficients for the iron losses - at this moment there is no function enabling import such data from another case solved earlier. For the purpose of the paper this additional computation time was about 10 minutes.

Determining steady-state values from transient data for each operation point is a common feature of machines with both the electrical circuit and permanent magnets. This topic was also analyzed by the authors in [11]. The importance of the problem is demonstrated on Fig. 3 and 4, representing the locked-rotor operation. This calculation test was performed for the purpose of obtaining fixed value of the starting torque. The characteristics on Figures 3-5 have been drawn for an interval shorter by half period on the start and at the end of the transient, because of the method of determining mean and rms values in the program. It can be noticed that the assumed computation time interval [0, 0.2] [s] is not sufficient to use obtained final values in optimization if a gradient routine is employed. Unbalance of the stator rms currents on Fig. 4 is a result of asymmetry of the rotor magnetic circuit. The accuracy of the computation results on this Figure is verified on Fig. 5.

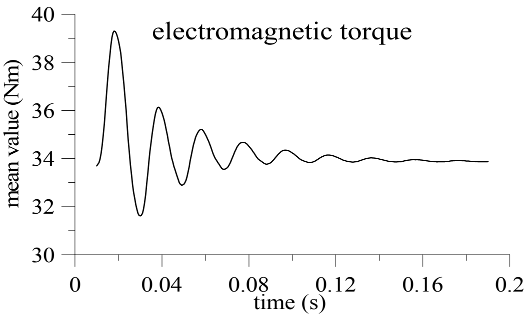

Fig. 3. Stepping mean value of the electromagnetic torque as a function of time for the locked-rotor operation.

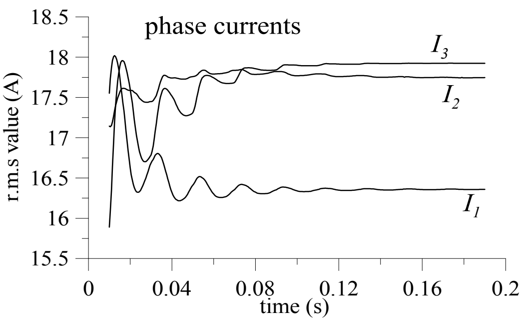

Fig. 4. Stepping rms value of stator phase currents as a function of time for the locked-rotor operation.

Computation time for the described case was about 108 minutes (400 steps Δt=0.0005s on a PC with Intel Core2 Duo E8400 processor), for the model with the highest number of nodes. A natural procedure in such a case seems to be an attempt to increase the time step in order to reduce the number of steps. Such a computation experiment has been performed for Δt=0.001s. As could be expected, time of computations decreased with a factor of about 2. Unfortunately, hard to accept differences in obtained values appeared on Fig. 2. The errors were equal to: ΔT = -0.5% for the electromagnetic torque, ΔI = -5.1% for rms stator current and ΔP_t = -14.8% for input power. These errors show that such a way of computation time reduction can not be accepted in optimization.

The above remarks justify an opinion that the total time of computation, with accuracy allowing obtained results to be used in an optimization procedure, is about 30 minutes for each case, in particular for a rotor position angle when the rated operation point and the breakdown E-M torque are determined.

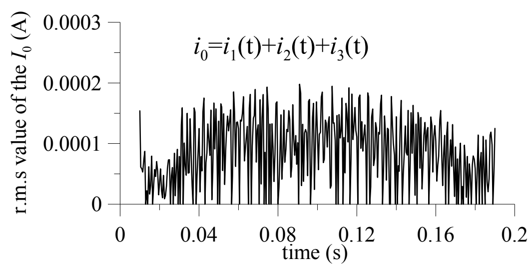

Fig. 5. Controlling the accuracy of currents computation on Fig. 4 - stepping rms value of the stator phase currents sum as a function of time for the locked-rotor operation.

4. Definition of a test optimization problem with the use of analytical model

An example of optimization calculations with an analytical model is used to estimate the time of corresponding calculations with a FEM model. A gradient optimization routine (a successive quadratic approximation method with BFGS formula of updating the Hessian) considered to be one of the most efficient is employed in this task. It is justified to expect that the number of objective function evaluations in the test will not be higher than in the case of a heuristic algorithm, a usual approach if FEM models are employed. The last remark is valid if standard software for optimization and FEM calculations is used.

An auxiliary scalar problem used when determining compromise solutions set of a bicriterial problem solved in [7] and defined in the form:

$$\max_{\mathbf{x}} \eta_N , \min_{\mathbf{x}} C_{ma} \bigg| \mathbf{x} \in X,\ \eta_N \geq 0.772 \qquad (1)$$

is under investigation.

Some properties of a solution of this auxiliary problem are given in Table 1, together with corresponding values for the initial design.

In the definition of Problem 1 the quantity $\boldsymbol{x}$ is a vector consisting of 15 optimization variables, X – the feasible region, η_N – rated efficiency, C_{ma} – cost of active materials. More information concerning this problem and its solution can be found in [7].

Table 1. Selected properties of the initial and optimal designs.

Property	Initial design	Optimal design
I_{1N} [A]	2.37	2.52
$cos\varphi_N$	0.974	0.907
η_N [%]	90.67	93.19
P_{fe} [W]	62.4	41.63
T_b [Nm]	25.6	20.4
T_l [Nm]	34.4	35.5
C_{ma} [PLN]	241.6	229.2

5. Estimation of a number of FEM calculation cases in optimization

A few papers concerning LSPMSM optimization with the use of FEM can be found in professional literature [1]-[3],[5]. In most of them FEM computations are only used to determine selected machine properties or parameters, and constitute a kind of a supplement for the lumped-parameter model [1],[2],[8]. Due to the lack of fully automated and proper quality procedure of searching for an extreme of an objective function with values determined in a FEM model, the attempts to find an optimal design solution for this model often terminate prematurely. As a consequence, the number of optimization variables is often reduced, and problem formulation simplified. A sensitivity analysis, performed usually at the very beginning of an optimization approach, is limited to one or two variables.

It can be noticed that solving an equivalent circuit in the analytical model provides a designer with similar information as one case of FEM calculations. The above solutions are necessary to determine all quantities required to define criterial functions and constraints in Problem 1. In particular, calculations have been performed for the rated operation load, the breakdown and locked-rotor E-M torques as well as conditions of a self-starting ability. The numbers of equivalent circuit calls related to all the above operation points have been determined in the test. The following results were obtained:

- The number of updates of Hessian of the objective function – 13.
- The number of calls of the objective function – 448.
- The total number of iterations required to determine the rated operation point – 3480 (Fig. 6).
- The total number of iterations required to determine the breakdown E-M torque– 2958 (Fig. 6).
- The number of iterations for the locked rotor state – 448.

The total number of iterations in the test was equal to 6886. According to earlier figures, it is equivalent to about 120 days of a continuous work of the above-mentioned PC, and it only concerns one scalar optimization task performed for one objective function and one feasible region definition. In practical applications it is necessary to repeat optimization calculations more than a dozen times for different variants of problem definition. In multi-dimensional objective and decision spaces it is the only way to gather knowledge about properties of objective and constraint functions. Often a multi-criterial approach is necessary if at least two criterial functions are not in accordance.

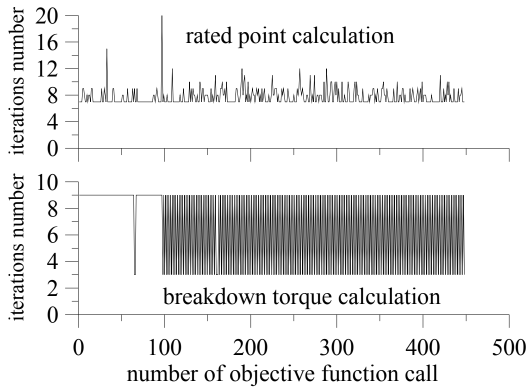

Fig. 6. The Number of iterations associated with obtaining the rated operation point (upper, and the breakdown E-M torque (lower) for subsequent calls of the objective function.

In the case of the analytical model, the total time of computations to solve Problem1 was about 192 [s] on a PC with Intel Core2 Quad 9550 processor (or Intel Core2 Duo E8400, time for both is very similar).

The above calculation times, for an analytical model (192[s]) and FEM model (120days), were estimated by an assumption, that in both cases the same task is performed. It is doubtful to find a remedy in a form of a heuristic algorithm for a problem with FEM model and 15 optimization variables. If a reduction of the number of these variables will be considered, it will be effective from the calculation time point of view, but obtained solution will be worse (a higher value of a minimum). Very likely a considerable reduction of calculation time of an optimization problem with a FEM model can be expected if a problem-oriented program is developed. Very likely it will not be a trivial, fast and cheap undertaking.

6. An influence of manufacturing accuracy and uncertainty of materials' properties.

There is a common opinion accepted by many people performing research works that the only credible approach to verify a model, analytical or FEM, is an experiment. Some issues related to this opinion are discussed in this paper.

The FEM models represent physical properties of machines with an accuracy, which is accepted in most applications. Apart from this, they have a valuable property that their results are repeatable, and can be used as reference quantities. Assuming that only relative quantities will be compared in an analysis, the FEM model results can be used as a representation of an existing motor.

In this paper the FEM results are applied to estimate an influence of a prototype manufacturing accuracy, as well as a discrepancy of physical properties of materials, on its operation data. The initial design data are assumed to be reference quantities and are denoted with a subscript "0". Calculations have been performed for the following cases:

- BH - iron magnetization curve $B(H)=0.9*B_0(H)$
- G1 - air gap width (g_0 +0.1mm),
- G2 - air gap width (g_0 - 0.1mm),
- B1 - remanent flux density of the permanent magnets $B_r=0.9*B_{r,0}$
- B2 - remanent flux density of the permanent magnets $B_r=1.1*B_{r,0}$
- Z1 – [(number of turns in a group) -1] (only in one coil section)
- Z2 – [(number of turns in a group) -3] (in each coil section - 1).

Such quantities as:

- electromagnetic torque Te
- rms value of the phase current Is
- stator iron losses originated by hysteresis PFe,hS
- eddy-currents losses PFe,eS
- total iron loss PFe,totS

are required in the optimization process when determining criterial and constraint functions, and for these quantities an analysis is performed.

Similarly as input data, the results of sensitivity analysis have been presented on Fig. 7 as a percentage difference of motor properties in relation to its initial design.

A remarkable influence of input design parameters on motor properties can be noticed for all cases. For instance, a 20% difference in the value of iron losses or 10% for the electromagnetic torque generate errors in determining rated operation data and efficiency, which can not be neglected. Such errors reduce the usefulness of the prototype in a verification process of the computation model considerably. It should be pointed out that:

a) the assumed discrepancies are significant but possible to occur (B2 seems to be less likely)
b) in practice it is hard or even impossible to establish these discrepancies without a destructive activity on the prototype
c) some material properties are changed by the technological process itself, and it is not an easy task to reveal to what extent.

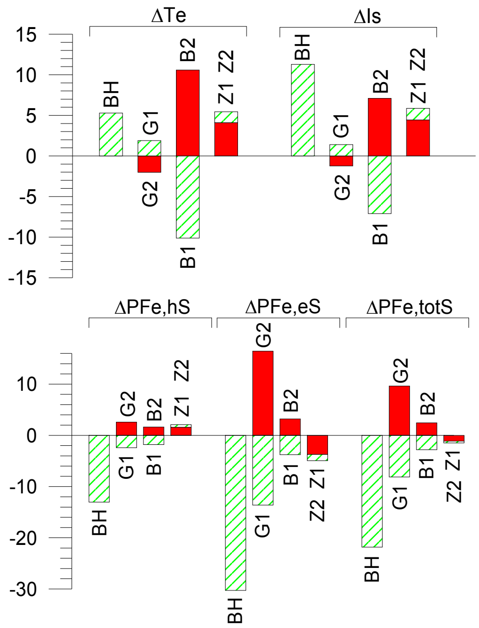

Fig.7 The influence of the inaccuracy of LSPMSM construction on its properties [%]. Description of the denotations are in the text.

Table 2. Errors resulting from differences BH, G1 i B2, in a relation to the initial design.

	Error [%]
Is [A]	17,5
Te [Nm]	19,6
PFe,hS	-13,4
PFe,eS	-36,0
PFe,totS	-24,9

Three discrepancies: BH, G1 and B2 have been imposed simultaneously in FEM computations

to estimate how much the prototype can differ from its design. Calculation results are presented in Table 2.
As could be expected, a superposition of errors occurred in the last case. In particular a difference of about 20% could be expected for the electromagnetic torque measured on the prototype. It would generate wrong conclusions about credibility of a model used in designing. Similarly, differences in power loss would influence the motor efficiency considerably and can produce a conclusion about a lack of credibility for the designing procedure.
Interesting information concerning troubles with verification of design calculations by means of a prototype can be found in [8]. Authors have obtained a better conformity in a comparison of the model to experimental data when the remanent flux density Br was less about 7% than it was assumed. It was within a tolerance of the permanent magnets manufacturer.

7. Conclusions

Detailed remarks regarding an application of the FEM and lumped-parameter models in optimization can be found in the paper. The most significant are as follows:

a) It arises from the performed analysis that a choice of a credible analytical model of the LSPMSM in optimization seems to be a reasonable decision.
b) Due to the expected very long time of computations, an attempt to employ FEM models in optimization will result in a simplification of the problem definition, a reduction of the corresponding research program, and as a consequence – a lower credibility of the final optimization results.
c) In general, an attempt to verify the analytical and FEM models by means of measurement data from a prototype produces wrong conclusions, as long as credible information about the machine construction (geometrical dimensions, winding parameters and materials' physical properties) can not be acquired; a proper identification procedure can be helpful in this task.
d) A properly developed FEM model fulfills requirements in a research work to be a credible reference for a corresponding analytical model.

8. References

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Reviewer

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