ANALYSIS OF THERMALLY UNSTEADY STATES OF ELECTRIC MACHINES AS STRUCTURES OF THE TWO BODIES

Abstract: When calculating the thermally unsteady state of electric machines, apart from the parameter on which depends the heating of machines in the thermally steady state there is still a parameter related to the thermal inertia of the individual parts of the machine. The aim of the paper is an analysis of the influence of the parameters affecting the thermal variations in the machine. The most convenient for this analysis is the assumption that the machine consists of two thermally inert elements because this assumption allows to perform the analysis analytically. The conclusions of the analysis also concern the machine treated as a system composed of many thermally inert elements.

As a result of the analysis a view has been obtained what role is played by time constants and what by thermal coefficients in heating and cooling of machines. It was shown that the time constants depend on that whether the machine rotates or not.

1. Introduction

A thermally unsteady state of an electric machine is such a state in which temperatures of the individual machine parts undergo changes. The cause of these changes is most often a change in losses emitted in the machine, whereby losses may change or not during changes in temperature. In the considerations it is assumed that losses do not change with time or linearly depend on machine temperatures. The dynamic states are characterized by thermal time constants.

An electric machine has a complex thermal structure thus its presentation by equations (differential) is complicated and solving a set of such equations is even more complicated. Two simplest cases will be discussed in order to make the considerations simple: (a) the machine is a single homogeneous body, (b) the machine is a structure of two thermally inert bodies, arbitrarily connected with each other and with the surroundings, such cases can be solved analytically and this makes it possible to analyse the thermal variations of the system.

2. Classical theory – a homogeneous body

The classical approach to the process of heating and cooling of a homogeneous body is based on the equation of energy equilibrium [2].

\[ \Delta P \, dt = mc \, \alpha \, d \theta + S \alpha (\theta - \theta_a) \, dt \] (1)

Where: \( \Delta P \) – power of losses given off in the considered body in [W], m- mass of body in [kg], c – specific heat of the body in [Ws/kg·K], S – area of the cooling surface of the body in \( m^2 \), \( \alpha \) – coefficient of heat transfer from the surface in \( W/m^2·K \), \( \theta \) – temperature of the body in \( ^\circ C \), \( \theta_a \) – the ambient temperature, \( d\theta \) increment of body temperature in the time \( dt \).

Assuming that \( m, c, d, \alpha \) and \( \theta_a \) are constant and the losses \( \Delta P \) do not change in the process of heating/cooling, the solution of equation (1) is obtained in the following form:

\[ \theta - \theta_a = \theta_s \left(1 - \exp\left(-\frac{t}{T}\right)\right) + \left(\theta_0 - \theta_a\right) \exp\left(-\frac{t}{T}\right) \] (2)

while the steady temperature rise is:

\[ \Delta \theta_s = \theta_s - \theta_a = \frac{\Delta P}{\alpha S} = R_\alpha \cdot \Delta P \] (3)

\( T \) – the time constant of the variation is equal to the product of the thermal inertia of the body and the thermal resistance between the body and the surroundings:

\[ T = \frac{mc}{\alpha S} = mc \cdot R_\alpha \] (4)

Formula (2) presents the general relationship of heating and cooling of a homogeneous body. The first expression of the formula concerns the heating of the body under the influence of constant losses \( \Delta P \), from the ambient temperature to the steady temperature rise. The second expression represents the variation of cooling of the body where no losses are given off from the temperature \( \theta_s \) (for \( t=0 \)) which the body had at the beginning of the thermal variation, till to the moment when the body gains the ambient temperature \( \theta_a \). The sum of the both exponential
expressions serves to calculate the temperature variation from the initial temperature \( \theta_0 \) (for \( t=0 \)) to the steady temperature \( \theta_s \). In the case of correspondingly high losses, when \( \theta_s > \theta_0 \), there follows the process of heating of the body, and when the losses are low, then \( \theta_s < \theta_0 \) and the body cools down. Formula (2) and its components are graphically shown in Fig.1.

Formula (2) serves both for calculating the curve of heating and cooling. However it should be noticed that cooling often occurs in different conditions than heating. An electric machine is usually heated up during motor operation that is during its rotation while it is cooling down when it is stopped. In higher than when the motor is stopped. It means that the time the state of motor rotation the coefficient of heat transfer is constant of the stopped machine is higher than in the state of rotation. It must be stressed that the time constant does not depend on it if the machine is heated or cooled but on it if the machine is stopped or if it rotates.

3. Thermal courses of the two inert bodies coupled to each other

The next degree of complexity of a machine thermal structure is two the existence of thermally inert bodies, which are arbitrarily connected to each other by means of thermal resistances. If the equivalent thermal network has nodes not having thermal inertia – then for a thermally unsteady state – this structure has still the character of two bodies. An exemplary equivalent schematic diagram of a system of two bodies is presented in Fig. 2 [1].

Comparing this schematic diagram with an electric machine it can be assumed that the stator winding represents one of those bodies and the stack of sheets or the other machine parts are the other body. It can be also assumed that the stator is one body and the rotor is the other one. In all cases, modeling the thermal structure by a system of two bodies will be a gross simplification. That schematic diagram has only this advantage that it can be solved analytically owing to which it is suitable for further analysis.

The energy balance of two thermally inert bodies arbitrarily connected, is expressed by a system of two differential equations:

\[
\frac{d\theta_1}{dt} + \xi_1 \theta_1 - \xi_1^* \theta_2 = \frac{\Delta P_{10}}{m_1 c_1} \\
\frac{d\theta_2}{dt} + \xi_2 \theta_2 - \xi_2^* \theta_1 = \frac{\Delta P_{2}}{m_2 c_2}
\]

(5)

in which the coefficients \( \xi \) are functions of thermal resistances of the between - node connections. For Fig. 2 these coefficients are equal to:

\[
\xi_1 = \frac{1}{m_1 c_1} \left( \frac{1}{R_1} - \beta \Delta P_{10} - \frac{R_3}{R_{13}} \right) \\
\xi_2 = \frac{1}{m_2 c_2} \left( \frac{1}{R_2} - \beta \frac{R_3}{R_{23}} \right) \\
\xi_1^* = \frac{1}{m_1 c_1} \left( \frac{1}{R_{12}} + \frac{R_3}{R_{13} R_{23}} \right) \\
\xi_2^* = \frac{1}{m_2 c_2} \left( \frac{1}{R_{12}} + \frac{R_3}{R_{13} R_{23}} \right)
\]

(6)

where \( R_1, R_2, \) and \( R_3 \) are the inverse of the sum of thermal conductivities contacting the node 1, 2 and 3.

E.g. \( 1/R_1 = 1/ R_{12} + 1/ R_{13} + 1/ R_{10} \), and \( \beta \) is the coefficient of the increase in losses as a function of temperature of the body (the winding).
After solving the equation set (5) we obtain the following temperature variation of thermally inert bodies with time:

\[
\begin{align*}
\theta_1 &= \theta_{1s} - K_1 e^{-\frac{t}{T_{K}}} - L_1 e^{-\frac{t}{T_{L}}} \\
\theta_2 &= \theta_{2s} - K_2 e^{-\frac{t}{T_{K}}} - L_2 e^{-\frac{t}{T_{L}}}
\end{align*}
\]

(7)

It is a biexponential variation with two time constants \(T_K\) and \(T_L\) \((T_K > T_L)\)

\[
\begin{align*}
\frac{1}{T_K} &= \frac{\xi_1 + \xi_2}{2} - \sqrt{\frac{(\xi_1 + \xi_2)^2}{4} + \xi_2^2 - \xi_1 \xi_2} \\
\frac{1}{T_L} &= \frac{\xi_1 + \xi_2}{2} + \sqrt{\frac{(\xi_1 + \xi_2)^2}{4} + \xi_2^2 - \xi_1 \xi_2}
\end{align*}
\]

(8)

The steady temperature rises are equal to:

\[
\begin{align*}
\Delta \theta_1 &= T_K T_L B_1 = T_K T_L \left[ \frac{\Delta P_{10}}{m_1 c_1} \xi_2 + \frac{\Delta P_{20}}{m_2 c_2} \xi_2 \right] = K_1 + L_1 + \theta_{10} \\
\Delta \theta_2 &= T_K T_L B_2 = T_K T_L \left[ \frac{\Delta P_{10}}{m_1 c_1} \xi_1 + \frac{\Delta P_{20}}{m_2 c_2} \xi_2 \right] = K_2 + L_2 + \theta_{20}
\end{align*}
\]

(9)

The temperature coefficients:

\[
\begin{align*}
K_1 &= \frac{B_T T^2}{T_K - T_L} - A_1 \\
K_2 &= \frac{B_T T^2}{T_K - T_L} - A_2 \\
L_1 &= A_1 - \frac{B_T T^2}{T_K - T_L} - \theta_{10} \\
L_2 &= A_2 - \frac{B_T T^2}{T_K - T_L} - \theta_{20}
\end{align*}
\]

(10)

\[
A_1 = \frac{T_K T_L}{T_K - T_L} \cdot \left[ \frac{\Delta P_{10}}{m_1 c_1} \xi_2 - \theta_{10} \left( \frac{1}{T_K} - \xi_2 \right) \right]
\]

(12)

\[
A_2 = \frac{T_K T_L}{T_K - T_L} \cdot \left[ \frac{\Delta P_{20}}{m_2 c_2} \xi_2 - \theta_{20} \left( \frac{1}{T_K} - \xi_1 \right) \right]
\]

It should be stressed that in these formulas the indices 1 and 2 concern the bodies 1 and 2 while the indices at the time constants \(T_K\) and \(T_L\) concern the whole system. It is not possible for one of the bodies to heat with other time constant than the other one, which would mean that one body reaches the steady thermal state while the other one is still heating up although they are thermally coupled to each other.

Fig. 3 presents the trends of heating curves calculated from formula (7) for the system of Fig. 2 with data shown below the figure. The curves denoted \(\Delta \theta_1\) and \(\Delta \theta_2\) present the temperature rise of bodies 1 and 2 of the network shown in Fig. 2. The components of those curves are equal to \(L \cdot \exp(-t/TL)\) and \(K \cdot \exp(-t/TK)\). After a time of \(4.6 \cdot TL\) the courses of temperatures \(\theta_1\) and \(\theta_2\) become single-exponential, shifted from the axis of abscises by \(L_1\) and \(L_2\).

4. Particular Cases

As it was be physically expected, the time constants depend only on thermal resistances and inertias. As losses of windings depend on temperatures there is a small correction of the time constants it as results of a change in \(\xi_i\). The steady temperature rises depend only on resistances and losses and the thermal inertia has no effect.

The calculated heating parameters (for \(\theta_a = 0\)) are shown in Fig. 4 for various values of thermal inertia \((mc)_2\) at constant values of losses of both bodies \(-P_f = P_2 = 0.3\ kW\). In the figure it can be seen that the coefficients \(K\) (corresponding to a longer time constant) are always positive, while the coefficient \(L\) has an sign opposite to coefficient \(L_2\) or they both are equal to zero. Physically this means that the losses of body corresponding to the exponential curve with a low time constant are flowing to a given body or leaving it.
Fig. 4 Thermal parameters for \( P_1 = P_2 = 0.3 \) kW, \((mc)_1=5\,[\text{Ws/K}]\), \( R_{12}=50\), \( R_{13}=200\), \( R_{23}=50\), \( R_{10}=70\), \( R_{30}=50\,[\text{K/kW}]\)

A. \( L_1 = L_2 = 0 \) at \( \theta_0 = 0 \)

When the following equation is satisfied:

\[
A_1 = \frac{B T_K T_L^2}{T_k - T_L}
\]

\[
A_2 = \frac{B T_K T_L^2}{T_K - T_L}
\]

then \( L_1 = L_2 = 0 \). For this example \( L_1 = 0 \) and \( L_2 = 0 \) for \((mc)_2 = 4.375\) at \( P_1 = P_2 = 0.3 \) kW (Fig. 4). It is interesting that although \( L_1 = L_2 = 0 \) the thermal time constant does nevertheless exist, because it does not depend on the distribution of losses at nodes of the equivalent network.

Fig. 5 Thermal parameters for \( P_1 = 0.1\) kW, \((mc)_1=5\), \( R_{12}=50\), \( R_{13}=200\), \( R_{23}=50\), \( R_{10}=70\), \( R_{30}=50\)

The variation of temperature of the both bodies is single – exponential although the system is composed of two bodies. For this case \( K_1 = \theta_1 \), and \( K_2 = \theta_2 \). The difference of temperatures between both bodies is equal:

\[
\theta_1 - \theta_2 = (\theta_{1s} - \theta_{2s}) - (K_1 - K_2) \cdot e^{-\frac{t}{K}}
\]  

(14)

Fig. 6 Thermal parameters for \( P_1 + P_2 = 0.6\) kW, \((mc)_1=5\), \( R_{12}=50\), \( R_{13}=200\), \( R_{23}=50\), \( R_{10}=70\), \( R_{30}=50\)

Thus it also changes single – exponentially. However for \((mc)_2 \neq 4.375\) the thermal coefficients \( L_1 \) and \( L_2 \) already play a role in the variation of the heating curve. Fig. 5 and Fig. 6 are calculated for the cases \( L_1 = L_2 = 0 \) whereby Fig. 5 is calculated for constant losses in body 2 while Fig. 6 for a constant sum of losses of the both nodes.

B. \( L_1 = L_2 = 0 \) at \( \theta_0 = 0 \) and \( K_1 = K_2 \)

When the distribution of losses at both the thermally inert nodes is such that the steady state temperature rises of both bodies are equal and at the same time \( L_1 = L_2 = 0 \) then \( K_1 = K_2 \). Parameters of such a thermal state are presented in Fig. 6 for \((mc)_2 = 2.6923\) and \( P_1 = 1.3 \) kW and \( P_2 = 0.7 \) kW. In this case there is no heat exchange between nodes 1 and 2 during the whole course of heating. This thermal state is obtained when the density of losses \( P/mc \) is similar for both bodies, thus it does not occur when one of the bodies is the stator winding, and the other one is the remainder of the machine, and the machine is loaded nominally.

C. Influence of \( \beta \) (\( \beta \neq 0 \))

The losses in windings vary depending on their temperature thus during heating at constant current these losses increase in the heating process. As it follows from formulae (6) and (8)
the time constants depend on the coefficient ($\beta$) of the increase in losses related to temperature change of windings, e.g. for $P_1 = P_2 = 1$ kW, $(mc)_1=5$, $(mc)_2=20$, $R_{12}=50$, $R_{13}=200$, $R_{23}=50$, $R_{10}=70$, $R_{30}=50$, $\beta=0.004$ the time constants changes from $T_K = 19.83 \text{ min}$ and $T_L = 118.19 \text{ sec}$ for $\beta=0$, to $T_K = 22.00 \text{ min}$ and $T_L = 129.13 \text{ sec}$ for $\beta=0.004$

5. Thermal courses of an electric machine presented in the form of many thermally inert bodies

The thermally unsteady state of electric machines is presented as an equivalent thermal network composed of many thermally inert elements (among which losses are given up in some of them) and of nodes which do not have the features of thermal inertia. For a system of $n$ thermally inert elements it is possible to form $n$ differential equations analogical to (5). It is not possible to solve such a system for $n>3$. This results from the fact that the first step is the solving of the following polynomial of the $n$ degree:

$$B_0 p^n + B_1 p^{n-1} + B_2 p^{n-2} + B_3 p^{n-3} ... + B_1 p^{n-1} + B_n p + B_n = 0$$

whose solution $p_1, p_2, ..., p_n$ is the inverse of the system’s time constants. The coefficients $B_0, ..., B_n$ of equation (13) are a function of thermal resistances and inertias of the system. The form of equation (13) testifies to the fact that the number of time constants is equal to $n$. Unfortunately, in case of a great number of network elements some time constants may not be very different from each other. A solution to this problem is thus more complex and will be presented the next time.

6. Conclusions

From these relations it can be concluded, that:

- There are not any time constants for a single part of a thermal system e.g. a winding or a core. There are time constants of the whole thermal structure of an electric machine. The often used notions of a time constant of winding and a time constant of core are not correct. From the physical point of view it is clear that it is not possible for one part of a thermal system to gain thermal equilibrium while the other one undergoes temperature changes. Only the influence of the particular time constants on heating the individual machine parts is different. This influence is expressed by the coefficients $K$ and $L$ and by their relation to the steady temperature rises.
- The time constants are the same, both for the process of heating as well as for cooling.
- The time constants (especially longer) depend on the coefficient $\beta$ that is they depend on losses given off in the windings.
- The time constants depend on thermal resistances, thus at higher thermal resistances (convection) which occur the when the electric machine does not rotate the time constants increase. Where here also it is indifferent whether the temperatures increase or decrease. It is only important if the machine rotates or it is stopped.

7. References


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